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Smart Manufacturing: Handling Preventive Actuator Maintenance and Economics Using Model Predictive Control

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Integrating components and systems of the manufacturing process is an important area of research to enable the future development and deployment of the Smart Manufacturing paradigm. An economic model predictive control (EMPC) scheme is proposed that effectively integrates scheduled preventive control actuator maintenance, process economics, and process control into a unified methodology. To accomplish this goal, a Lyapunov-based EMPC (LEMPC) scheme is formulated for handling changing number of online actuators (i.e., changing number of manipulated inputs). Closed-loop stability under the proposed LEMPC is proven. Subsequently, the LEMPC is applied to a chemical process network used for benzene alkylation to demonstrate that the LEMPC can maintain stability and improve dynamic economic performance of the process network in the presence of changing number of available control actuators resulting from scheduled preventive maintenance tasks. *AIChE J*, 60: 2179–2196, 2014
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Introduction

Smart Manufacturing has been deemed by many manufacturing experts as the next frontier of manufacturing that will revolutionize future manufacturing.[1–5] Objectives put forth by the proponents of Smart Manufacturing can be summarized as the design, development, and deployment of integrated systems to achieve a significant step change in overall manufacturing intelligence.[2,4] One example of the challenge problems next-generation manufacturing faces is being able to successfully manage the trade-off between sustainability and profitability.[5] In the current manufacturing paradigm, many individual components or phases of the manufacturing process are optimized and/or operated independently of other components (e.g., planning/scheduling, control, plant-wide optimization etc.) and thus, the aforementioned challenge problem cannot be handled in the context of traditional manufacturing paradigms. One of the key components of Smart Manufacturing is to unite individual components into a completely integrated platform.[4] The results of making these interconnections have the potential to transform operations from a reactive or corrective environment to a proactive or preventive setting yielding major economic benefit.[1] Identifying the interconnections between components and systems has been the subject of recent research.

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Specifically, in the context of process operations, maintenance programs and policies are a vital part of maintaining operations, reliability, and safety of manufacturing processes (e.g., Refs. 6, 7). Maintenance tasks can be divided into two main categories: (1) corrective and (2) preventive.[8] Corrective maintenance deals with repairing or replacing a failed component of the process; while preventive maintenance consists of tasks or measures taken to prevent component failure such as routine inspection of components for defects or excess wear and refurbishment of components. The scope and scale of the latter maintenance program varies in the process industries. At one end of the scale, scheduled preventive maintenance may only consist of a spreadsheet containing a schedule and historical log of the preventive maintenance tasks which has been compiled from past experience. Another simple approach to preventive maintenance could be to utilize existing process identification tools or alarms used to assess safety and operability performance of processes. For example, preventive maintenance action may be taken when tools identify near misses which are considered to be precursors to abnormal events.[9] Conversely, more complex, model-based, and optimization-based approaches to preventive maintenance have been explored (e.g., Refs. 6,8,10–12). In fact, extensive literature exists on the mathematical theory of reliability which is a key metric in most complex preventive maintenance programs.[6] Examples of optimization-based approaches to preventive maintenance include: developing a framework for preventive maintenance optimization to solve the so-called opportunistic maintenance problem by combining Monte Carlo simulation with a

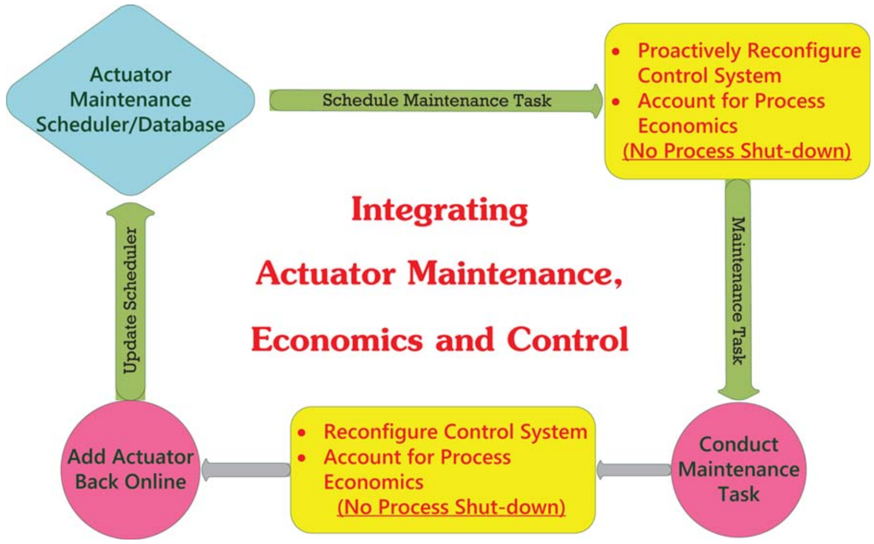

Figure 1. Integrated approach to preventive control actuator maintenance, process control, and real-time economic process performance optimization.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

genetic algorithm,[12] integrating statistical process control techniques with optimization of preventive maintenance policies which was demonstrated to yield a reduction in operation costs over only using a control chart or preventive maintenance policy,[10] determining the optimal maintenance policy by constructing an optimization problem which includes maximizing the expected revenue minus the maintenance costs subject to a (steady-state) process model and maintenance model,[8] and developing an integrated model to coordinate maintenance planning and production scheduling.[11]

Preventive maintenance is of interest in the context of the Smart Manufacturing given the direct connection between the objectives of the two. Furthermore, maintenance costs can be significant. In the context of the literature on maintenance policies, an often cited statistic on maintenance costs is that it may comprise of up to 20–30% of the operating budget of a chemical plant,[13] and therefore, improving maintenance practices can impact maintenance costs and production losses.[12] Developing so-called "Smart" maintenance policies/systems through unifying manufacturing components especially those that reduce process upset, loss, and downtime like preventive maintenance programs is an important task given the possibility of significant cost-savings.

A high percentage of the small-scale, day-to-day preventive maintenance tasks for the chemical process industry are for control actuators of process control systems (e.g., compressors, pumps, control valves etc.).[12] To accomplish these preventive maintenance tasks, the ability for a control system to maintain stable operation of the process while dictating an economically optimal operating policy with respect to the available control actuators is desirable and can be considered within the scope of the Smart Manufacturing paradigm. One natural framework that can be extended to accomplish this task is to use economic model predictive control (EMPC). EMPC is an optimization-based control technique that optimizes economic process performance over a (control) horizon by using a dynamic process model to predict the evolution of the process.[14–23] Some of the recent developments on EMPC include: proving asymptotic stability of EMPC formulated without terminal constraints,[18] proposing an EMPC scheme with self-tuning terminal cost,[22] and formulating an EMPC that can account for explicitly time-varying parameters in the cost function.[17] EMPC was first presented as a control methodology to overcome some of the challenges faced with integrating real-time optimization (RTO) and regulatory control, but a consequence of the unique formulation of EMPC (i.e., control methodology that accounts directly for the process economics) is that it can be integrated into other systems as part of the Smart Manufacturing paradigm (e.g., preventive maintenance programs). However, EMPC cannot be applied directly because the optimization problem dimensionality, cost function, and constraints change as a result of the changing number of inputs.

Utilizing a pre-existing preventive maintenance schedule or policy, the task of accounting for scheduled actuator maintenance via the control system is considered. Specifically, the focus of this work is to develop a Lyapunov-based EMPC (LEMPC) method that can maintain stability while also, dictate an economically optimal dynamic operating policy with changing number of manipulated inputs as a result of control actuators being taken offline for preventive maintenance or placed back online after the maintenance work has been completed. To deal with the closed-loop stability challenge, we extend our previous work on integrating traditional control (e.g., model predictive control with quadratic cost function) with preventive maintenance[24] to EMPC. We formulate and prove stability for a LEMPC scheme capable of handling these objectives. The overall approach effectively integrates and closes the loop around actuator maintenance, real-time process economic optimization, and feedback control which is summarized in the closed-loop diagram of Figure 1. The proposed LEMPC is applied to a process network used for the alkylation of benzene to demonstrate that the LEMPC is able to maintain stability of the process, perform successful reconfiguration of the control system accounting for variable number of manipulated inputs, and operate the process in an economically optimal fashion.

Preliminaries

Notation

The following notation will be used in this work. The operator $|\cdot|$ denotes the Euclidean norm of a vector. A continuous function $\alpha:[0,a)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and is equal to zero when evaluated at zero (i.e., $\alpha(0)=0$). A level set (level surface) of a scalar function $V:\mathbb{R}^n \to \mathbb{R}$ is denoted as $\Omega_r := \{x \in \mathbb{R}^n : V(x) \leq r\}$. The symbol diag (v) denotes a diagonal matrix with diagonal elements equal to the elements of the vector v.

Class of nonlinear process systems

In the present work, the class of nonlinear process systems considered for the design of an EMPC scheme for handling preventive maintenance of the jth control actuator are of the form

$$\dot{x}(t)=f(x(t),u(t),w(t)) \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the input vector, and $w \in \mathbb{R}^l$ is the disturbance vector. The amount of available control energy is bounded in a convex set. A scheduler or decision-maker schedules a preventive maintenance task on the jth actuator that effectively takes the jth actuator offline at t_r. In other words, the set of available control actions with all actuators online is given by

$$U_0=\{u \in \mathbb{R}^m : |u_i| \leq u_i^{\max}, i=1,\ldots,m\}$$

for $t \in [t_0, t_r)$. After the jth actuator is taken offline at t_r, the set becomes

$$U_j=\{u \in \mathbb{R}^m : |u_i| \leq u_i^{\max}, i=1,\ldots,j-1,j+1,\ldots,m, u_j=0\}$$

until the maintenance task is completed, and the jth actuator is brought back online at t'_r. The vector function f is assumed to be a locally Lipschitz vector function of its arguments. The disturbance vector is considered to be bounded in a set, that is

$$W=\{w \in \mathbb{R}^l : |w| \leq \theta\}$$

where $\theta > 0$ bounds the norm of the disturbance vector w. The state vector of the continuous-time system of Eq. 1 is assumed to be measured at sampling instances: $t_k=t_0+k\Delta, k = 0,1,\ldots$ where Δ is the sampling period and $t_0=0$ (without loss of generality).

With respect to the system of Eq. 1, a time-invariant cost function that describes the real-time process economics is assumed to be available of the form $l_e(x,u)$. The economically optimal steady-state for the system with respect to the economic cost for the system with all available actuators online is denoted as $i=0$ and for the system with the jth actuator offline is denoted as $i=j$ and are defined as

$$x_{s,i}^* = \arg\max_{x_s \in X_s} \{l_e(x_s,u_s) : f(x_s,u_s,0)=0, u_s \in U_i\}, \quad i=0,j \tag{2}$$

where the set X_s is the set of admissible steady-states.

Remark 1. *It is important to note that the present work focuses on control actuator maintenance. Handling sensor maintenance is not within the scope of the present work as other issues must be considered (e.g., state estimation, observability, etc.).*

Stabilizability assumption

Additional assumptions must be placed on the system of Eq. 1 to guarantee that the closed-loop system can be stabilizable with all available control actuators and with $m-1$ control actuators (i.e., the jth actuator is offline). Specifically, the existence of two explicit Lyapunov-based controllers $h_i(\bar{x}_i)$ for $i=0,j$ that render the steady-state $x_{s,i}^*$ of the nominal system of Eq. 1 asymptotically stable under continuous implementation is assumed. The notation $\bar{x}_i$ is the deviation of the state from the corresponding steady-state (i.e., $\bar{x}_i=x-x_{s,i}^*$). Using converse theorems,[25,26] the existence of continuous differentiable Lyapunov functions $V_i(\bar{x}_i)$ for $i=0,j$ for the closed-loop system with all m actuators and with $m-1$ actuators, respectively, follows from the stabilizability assumption. The closed-loop Lyapunov functions under the Lyapunov-based controllers satisfy the following conditions

$$\alpha_{1,i}(|\bar{x}_i|) \leq V_i(\bar{x}_i) \leq \alpha_{2,i}(|\bar{x}_i|) \tag{3a}$$

$$\frac{\partial V_i(\bar{x}_i)}{\partial x} f(\bar{x}_i+x_{s,i}^*, h_i(\bar{x}_i), 0) \leq -\alpha_{3,i}(|\bar{x}_i|) \tag{3b}$$

$$\left|\frac{\partial V(\bar{x}_i)}{\partial x}\right| \leq \alpha_{4,i}(|\bar{x}_i|) \tag{3c}$$

$$h_i(\bar{x}_i) \in U_i, \quad i=0,j \tag{3d}$$

for $\bar{x}_i \in D_i$ where D_i is an open neighborhood of the origin for $i=0,j$. This assumption is similar to assuming that the pair (A,B) is stabilizable for linear systems. The stability regions $\Omega_{\rho_i} \subseteq D_i$ for $i=0,j$ can be estimated for the closed-loop system of Eq. 1 under the explicit stabilizing controllers $h_0(x-x_{s,0}^*)$ and $h_j(x-x_{s,j}^*)$ by taking these regions to be a level set of the Lyapunov function where the Lyapunov function is decreasing along the closed-loop state trajectory. As Ω_{ρ_i} is taken to be a level set of V_i, it is a compact (closed and bounded) set. For the remainder of this work, a state x is said to be contained in the set Ω_{ρ_i} if the deviation state $(\bar{x}_i=x-x_{s,i}^*)$ is contained in Ω_{ρ_i}. A variety of control laws have been developed using Lyapunov techniques for various classes of nonlinear systems (see Refs. 27,28 and the references therein) that allow for an explicit characterization of the stability region Ω_{ρ_i} while accounting for input constraints and thus, the offline computation required to compute Ω_{ρ_i} may be insignificant for some cases.

Additionally, certain conditions must be satisfied to ensure that it is possible to force the closed-loop state from any point in the stability region Ω_{ρ_0} to the stability region Ω_{ρ_j} in preparation for taking the jth control actuator offline for maintenance. These conditions must also ensure that it is possible to force the closed-loop state back to the stability region Ω_{ρ_0} from any point in Ω_{ρ_j} after the maintenance task is completed and the jth actuator is ready to be placed back online. To establish these conditions, the closed-loop properties of the Lyapunov-based controllers applied in a sample-and-hold fashion to the system of Eq. 1 are presented below.

First, some basic properties of the closed-loop system are needed. By the Lipschitz property of the vector field f, the continuous differentiability property of the Lyapunov function, and the compactness of Ω_{ρ_i}, there exist positive constants $L_{x,i}, L_{w,i}, L'_{x,i}$, and $L'_{w,i}$ for $i=0,j$ such that the following inequalities hold

$$|f(x,u,w)-f(x',u,0)| \leq L_{x,i}|x-x'|+L_{w,i}|w| \tag{4}$$

$$\left|\frac{\partial V_i}{\partial x}f(x,u,w)-\frac{\partial V_i}{\partial x}f(x',u,0)\right| \le L'_{x,i}|x-x'|+L'_{w,i}|w| \tag{5}$$

for all $(x-x^*_{s,i}),(x'-x^*_{s,i}) \in \Omega_{\rho_i}, u \in U_i, w \in W$ for $i=0,j$. By continuity, the bound on the inputs and the aforementioned Lipschitz properties, a positive constant M_i can be found to bound the vector field

$$|f(x,u,w)| \le M_i \tag{6}$$

that holds for all $(x-x^*_{s,i}) \in \Omega_{\rho_i}, u \in U_i, w \in W$ for $i=0,j$.

The main stability result of applying a Lyapunov-based controller in a sample-and-hold fashion in the presence of disturbances is provided below without proof for the sake of brevity. The interested reader is referred to Ref. 29 for a complete discussion and proof of the main result on the Lyapunov-based controller. The following proposition establishes practical stability of the Lyapunov-based controller when applied to the system of Eq. 1.

Proposition 1 (c.f. Ref. 29). *Consider the closed-loop system of* Eq. 1 u*nder the controller $h(\bar{x}_i)$ that satisfies the conditions of* Eq. 3 *when the controller $h(\bar{x}_i)$ is implemented in a sample-and-hold fashion with sampling period $\Delta > 0$. Let $\Delta > 0, \rho_i > \rho_{s,i} > 0, \rho_{\min,i} \le \rho_i$, and $\epsilon_{s,i} > 0$ satisfy*

$$-\alpha_{3,i}(\alpha_{2,i}^{-1}(\rho_{s,i}))+L'_{x,i}M_i\Delta+L'_{w,i}\theta \le -\epsilon_{s,i}/\Delta \tag{7}$$

for each $i=0,j$. Then, the Lyapunov function $V_i(x-x^*_{s,i})$ will decrease over the sampling period Δ

$$V(x(t_{k+1})-x^*_{s,i}) \le V(x(t_k)-x^*_{s,i})-\epsilon_{s,i} \tag{8}$$

for any $(x(t_k)-x^*_{s,i}) \in \Omega_{\rho_i} \backslash \Omega_{\rho_{s,i}}$. Furthermore, if $(x(0)-x^*_{s,i}) \in \Omega_{\rho_i}$, then $(x(t)-x^*_{s,i})$ is ultimately bounded in $\Omega_{\rho_{\min,i}}$ where

$$\rho_{\min,i} := \max_{\tau\in[t_k,t_k+\Delta)} \left\{V_i(x(\tau)-x^*_{s,i}) : V_i(x(t_k)-x^*_{s,i}) \le \rho_{s,i}\right\} \tag{9}$$

Under the Lyapunov-based controller, the closed-loop system will converge to a neighborhood of the steady-state for a sufficiently small sampling period and bound on the disturbance. Applying Eq. 8 recursively, one can show that the state converges to $\Omega_{\rho_{s,i}}$ in a finite number of sampling periods. Once the state converges to $\Omega_{\rho_{s,i}}$, it is maintained in $\Omega_{\rho_{\min,i}}$ as a result of the definition of $\Omega_{\rho_{\min,i}}$ (Eq. 9). This implies that one can find a sufficiently long (finite time) horizon such that the closed-loop state will converge to $\Omega_{\rho_{\min,i}}$ by the end of the horizon for any initial state $(x(t_0)-x^*_{s,i}) \in \Omega_{\rho_i}$. With the stability properties of the Lyapunov-based controllers, conditions are imposed on $x^*_{s,j}$ which will be used in the design of an EMPC for handling actuator maintenance and is stated in the following assumption. Assumption 1 can be satisfied by imposing appropriate constraints in the steady-state optimization problem constructed to solve for $x^*_{s,j}$.

Assumption 1. *The steady-state $x^*_{s,j}$ is chosen so there exists a region $\Omega_{\rho^*_j}$ with $\rho^*_j \ge \rho_{\min,j}$ such that $\Omega_{\rho^*_j} \subseteq \Omega_{\rho_0}$. Furthermore, the stability region Ω_{ρ_j} contains a region $\Omega_{\rho^*_0}$ where $\rho^*_0 \ge \rho_{\min,0}$.*

Applying the results of Proposition 1 and the conditions of Assumption 1, a sufficiently long operating horizon denoted as $t_{N^*_0}$ can be found such that the closed-loop state can be forced to Ω_{ρ_j} starting from any initial state in Ω_{ρ_0} because the closed-loop state can be forced into $\Omega_{\rho^*_0} \subseteq \Omega_{\rho_j}$ in a finite number of sampling periods. Similar arguments can be applied to define a sufficiently long horizon $t_{N^*_j}$, such that any initial state in the stability region Ω_{ρ_j} can be forced into the stability region Ω_{ρ_0} by the end of the horizon. Thus, an operating horizon t_{N^*} defined as

$$N^* = \max\left\{N^*_0, N^*_j\right\} \tag{10}$$

can be found which is a horizon that guarantees that the closed-loop state under the Lyapunov-based controller $h_0(x-x^*_{s,0})$ implemented in a sample-and-hold fashion satisfies $(x(t_{N^*})-x^*_{s,j}) \in \Omega_{\rho_j}$ for any initial state $(x(0)-x^*_{s,0}) \in \Omega_{\rho_0}$, and for all initial states $(x(0)-x^*_{s,j}) \in \Omega_{\rho_j}$, the closed-loop state under the Lyapunov-based controller $h_j(x-x^*_{s,j})$ implemented in a sample-and-hold fashion satisfies $(x(t_{N^*})-x^*_{s,j}) \in \Omega_{\rho_0}$.

Remark 2. *The main idea of accounting for control actuator maintenance in the control system is to maintain operation of the process while an actuator is taken offline to be repaired or replaced. To accomplish this, the control system must be able to first force the process into a region where the $m-1$ remaining actuators can maintain stability of the system and is robust to the influence of disturbances and uncertainty. The next priority would be to operate the process in an economically optimal way with the $m-1$ available actuators. Assumption 1, while it may restrict the feasible set of the admissible steady-states for computing the economically optimal steady-state with $m-1$ actuators, reflects this hierarchical level of objectives of the control system. Instead of imposing Assumption 1, one could assume the existence of an input trajectory that can force the closed-loop state from the stability region Ω_{ρ_0} to the stability region Ω_{ρ_j} and then, force the state back to Ω_{ρ_0} after the maintenance is completed. However, this assumption is difficult, in general, to verify.*

Lyapunov-based EMPC

Utilizing the properties presented above, a LEMPC inspired by the results in Ref. 19 will be used to design an EMPC scheme that explicitly accounts for control actuator maintenance. A brief review of the formulation of LEMPC is provided below. LEMPC is characterized by the following optimization problem

$$\max_{u\in S(\Delta)} \int_{t_k}^{t_{k+N}} l_e(\tilde{x}(\tau),u(\tau))d\tau \tag{11a}$$

$$\text{s.t. } \dot{\tilde{x}}(t)=f(\tilde{x}(t),u(t),0) \tag{11b}$$

$$\tilde{x}(t_k)=x(t_k) \tag{11c}$$

$$u(t) \in U, \forall t \in [t_k, t_{k+N}) \tag{11d}$$

$$V(\tilde{x}(t)) \le \tilde{\rho}, \forall t \in [t_k, t_{k+N}) \quad \text{if } V(x(t_k)) < \tilde{\rho} \tag{11e}$$

$$\frac{\partial V}{\partial x}f(x(t_k),u(t_k),0) \le \frac{\partial V}{\partial x}f(x(t_k),h(x(t_k)),0), \quad \text{if } V(x(t_k)) \ge \tilde{\rho} \tag{11f}$$

where the decision variable of the optimization problem is the piecewise constant input trajectory over the finite-time prediction horizon t_k to t_{k+N}. Within this context, the economic cost function, which describes the process economics, is optimized over the finite prediction horizon, $S(\Delta)$ denotes the family of piecewise constant functions and $\tilde{x}$ denotes the predicted state trajectory of the system with the input trajectory computed by the LEMPC.

To predict the state trajectory under the computed input trajectory, the nominal dynamic model of the process system is used (Eq. 11b) with an initial condition provided by a measurement of the current state (Eq. 11c). The constraint of Eq. 11d is the bound on the available control actuation. The remaining two Lyapunov-based constraints are used to ensure that the closed-loop state is always maintained in the stability region Ω_ρ and defines a two mode control strategy. Mode 1 operation of the LEMPC is active (i.e., the constraint of Eq. 11e is imposed on the optimization problem) when the current state is within $\Omega_{\tilde{\rho}}$ where $\Omega_{\tilde{\rho}}$ is a subset of the stability region Ω_ρ. The size of $\Omega_{\tilde{\rho}}$ is a function of the stability properties of the system. Under Mode 1 operation of the LEMPC, the LEMPC allows for dynamic operation to optimize the economic cost function while maintaining the predicted evolution in $\Omega_{\tilde{\rho}}$. If the current state $x(t_k) \in \Omega_\rho \backslash \Omega_{\tilde{\rho}}$, Mode 2 is active (i.e., the constraint of Eq. 11f is imposed). Under Mode 2 operation of the LEMPC, the LEMPC optimizes the input trajectory with respect to the economic cost while enforcing that the computed control action decreases the Lyapunov function value by at least the rate given by the Lyapunov-based controller for the first sampling period of the prediction horizon. With the two Lyapunov-based constraints, stability under LEMPC is defined as maintaining the closed-loop state in the stability region Ω_ρ (i.e., the set Ω_ρ is an invariant set for the closed-loop system under LEMPC) and is guaranteed for any initial state $x(t_0) \in \Omega_\rho$ (see Ref. 19 for details on this point).

Remark 3. *Without loss of generality, the origin of the unforced system (i.e., $f(0,0,0)=0$) is assumed to be the equilibrium of the model of Eq. 11b and thus, the Lyapunov-based constraints are formulated with a Lyapunov function with respect to the origin. The origin is typically also taken to be the economically optimal steady-state. Specifically, in conventional model predictive control (MPC) schemes and in EMPC formulated with a terminal constraint, the target state or set point that is used in these MPC schemes is usually the economically optimal steady-state. In LEMPC, the Lyapunov-based constraints of Eqs. 11e and 11f do not necessarily need to be formulated with the economically optimal steady-state because the LEMPC may enforce a dynamic operating policy that is better than operating at the economically optimal steady-state. Therefore, the Lyapunov-based constraints could be formulated with a Lyapunov function for some other steady-state. This steady-state can be chosen, for instance, as a steady-state that yields a large estimate of the stability region or a steady-state whose corresponding stability region is a region in state space where process constraints (e.g., input and state constraints) are satisfied. Throughout the theoretical developments of an LEMPC scheme for handling control actuator maintenance, the economically optimal steady-states (i.e., $x^*_{s,0}$ when all m actuators are available and $x^*_{s,j}$ when $m-1$ actuators are available) will be used for the sake of consistency between LEMPC and EMPC formulated with a terminal constraint where the terminal constraint is typically taken to be the economically optimal steady-state.*[15]

Proposed LEMPC Scheme for Handling Actuator Maintenance

In this section, the design of the LEMPC for explicitly handling control actuator maintenance is presented. First, the implementation strategy and formulation of the proposed LEMPC is provided. Subsequently, the main theoretical contribution of this work is given.

Implementation and formulation

The implementation strategy is similar to the implementation strategy of LEMPC[19] that does not handle actuator maintenance. The main difference in the implementation strategy occurs during the period of time when the LEMPC proactively transitions from the control configuration with all available m actuators to the control configuration with $m-1$ actuators before the jth actuator is taken offline and vice versa when the actuator is brought back online after the maintenance task is completed. Much like Ref. 19, subsets of the stability region whereby dynamic operation is allowed are defined to make the sets Ω_{ρ_0} and Ω_{ρ_j} invariant in the presence of disturbances and uncertainties. The two sets are denoted $\Omega_{\tilde{\rho}_0} \subset \Omega_{\rho_0}$ and $\Omega_{\tilde{\rho}_j} \subset \Omega_{\rho_j}$ and are explicitly characterized in the "Closed-loop stability analysis" subsection below. The main advantage of using EMPC (LEMPC) over other stabilizing controllers is to take advantage of the unique ability of EMPC to optimize dynamic operation (inherently transient) with respect to the process economics during the transition to the new control configuration.

The actuator is to be taken offline at t_r which has been scheduled by a maintenance scheduler. The actuator will be put back online at t'_r after the maintenance task has been completed and the actuator is ready to be put back online. Prior to t_r and t'_r, it is desirable from a stability point-of-view to transition to the next control configuration (i.e., m available actuators to $m-1$ actuators and vice versa). To do this, the stability region of the next control configuration is used in the formulation of the LEMPC. More specifically, the state must converge to the set Ω_{ρ_j} by t_r and be maintained in Ω_{ρ_j} for $t \in [t_r, t'_r)$. Similarly, the state must converge to the set Ω_{ρ_0} by t'_r.

To accomplish the above control objectives, the proposed LEMPC scheme for handling control actuator maintenance (i.e., taking the jth control actuator offline at t_r) is as follows

$$\max_{u \in S(\Delta)} \int_{t_k}^{t_{k+N}} l_e(\tilde{x}(\tau), u(\tau))d\tau \tag{12a}$$

$$\text{s.t. } \dot{\tilde{x}}(t) = f(\tilde{x}(t), u(t), 0), \quad \tilde{x}(t_k) = x(t_k) \tag{12b}$$

$$u(t) \in U_0, \ \forall t \in [t_k, t_r) \tag{12c}$$

$$u(t) \in U_j, \ \forall t \in [t_r, t_{k+N}) \tag{12d}$$

$$\begin{aligned} &V_0(\tilde{x}(t) - x^*_{s,0}) \le \tilde{\rho}_0, \ \forall t \in [t_k, t_{k+N}) \\ &\text{if } V_0(x(t_k) - x^*_{s,0}) < \tilde{\rho}_0 \ \text{ and } \ t_r \notin [t_k, t_{k+N}) \end{aligned} \tag{12e}$$

$$\begin{aligned} &\frac{\partial V_0}{\partial x} f(x(t_k), u(t_k), 0) \le \frac{\partial V_0}{\partial x} f(x(t_k), h_0(x(t_k) - x^*_{s,0}), 0), \\ &\text{if } (V_0(x(t_k) - x^*_{s,0}) \ge \tilde{\rho}_0 \ \text{ and } \ t_r \notin [t_k, t_{k+N})) \\ &\text{or } (V_j(x(t_k) - x^*_{s,j}) \ge \rho_j \ \text{ and } \ t_r \in [t_k, t_{k+N})) \end{aligned} \tag{12f}$$

where the notation is similar to that of the LEMPC of Eq. 11. When $t_r \notin [t_k, t_{k+N})$, the Lyapunov-based constraints that define Mode 1 and Mode 2 operation of the LEMPC are identical to the constraints used in the LEMPC of Eq. 11. If the time the actuator is taken offline is within the prediction horizon (i.e., $t_r \in [t_k, t_{k+N})$) and the current state is outside Ω_{ρ_j}, the Mode 2 constraint is active to force the closed-loop state closer to $\Omega_{\rho_0^*} \subseteq \Omega_{\rho_j}$. Once the state converges to the region Ω_{ρ_j}, the Lyapunov-based constraints of the LEMPC switches to

$$\max_{u \in S(\Delta)} \int_{t_k}^{t_{k+N}} l_e(\tilde{x}(\tau), u(\tau)) d\tau \tag{13a}$$

$$\text{s.t. } \dot{\tilde{x}}(t) = f(\tilde{x}(t), u(t), 0), \quad \tilde{x}(t_k) = x(t_k) \tag{13b}$$

$$u(t) \in U_0, \ \forall t \in [t_k, t_r) \tag{13c}$$

$$u(t) \in U_j, \ \forall t \in [t_r, t_{k+N}) \tag{13d}$$

$$V_j(\tilde{x}(t) - x^*_{s,j}) \leq \tilde{\rho}_j, \ \forall t \in [t_k, t_{k+N}) \quad \text{if } V_j(x(t_k) - x^*_{s,j}) < \tilde{\rho}_j \tag{13e}$$

$$\frac{\partial V_j}{\partial x} f(x(t_k), u(t_k), 0) \leq \frac{\partial V_j}{\partial x} f(x(t_k), h_j(x(t_k) - x^*_{s,j}), 0), \quad \text{if } V_j(x(t_k) - x^*_{s,j}) \geq \tilde{\rho}_j \tag{13f}$$

based on the Lyapunov function and the Lyapunov-based controller with $m-1$ control actuators. However, the LEMPC can utilize all m actuators until t_r when the jth actuator is taken offline. In this fashion, the actuator maintenance is proactively accounted for via the control system. Owing to the fact that the control configuration is switching from m available actuators to $m-1$ available actuators, the LEMPC of Eq. 13 will be feasible if the closed-loop state converges to the region Ω_{ρ_j} before t_r because a solution with $u_j(t) = 0$ for $t \in [t_k, t_r)$ is a feasible solution to the optimization problem (this point will be discussed further in the "Closed-loop stability analysis" subsection below).

Once the actuator is ready to be brought back online, the state must converge to the set Ω_{ρ_0} by t'_r. The LEMPC for this case is similar to Eq. 12 until the closed-loop state converges to Ω_{ρ_0} when the LEMPC switches to a formulation similar to Eq. 13. The formulations for each of these phases of operation are the same as Eqs. 12–13 except for the following notation modifications: $0 \rightarrow j, j \rightarrow 0$, and $t_r \rightarrow t'_r$. Another difference in the implementation strategy when switching from $m-1$ available actuators to m available actuators is that after the closed-loop state has converged to Ω_{ρ_0} no guarantee can be made that $m-1$ actuators can maintain the closed-loop state in Ω_{ρ_0}. However, this presents little practical complications as the time t'_r most likely can be treated as a soft constraint. In other words, t'_r can be treated as the time the closed-loop state converges to Ω_{ρ_0} because the transition from $m-1$ actuators to m actuators will likely only be activated after the maintenance task has been successfully completed. If the time t'_r is a hard constraint, one could force the LEMPC to operate in Mode 2 (based on the Lyapunov function and Lyapunov-based controller for $m-1$ actuators) to enforce the system to converge to $\Omega_{\rho_j^*} \subseteq \Omega_{\rho_0}$ and maintain it there until t'_r when the actuator is ready to be brought back online.

The proposed LEMPC scheme for handling scheduled control actuator maintenance (consisting of switching constraints) is implemented in a receding horizon fashion. The optimal solution, obtained at each sampling period, to the optimization problem (either Eq. 12 or Eq. 13 depending on the phase of operation) is denoted as $u^*(t)$ which is defined for $t \in [t_k, t_{k+N})$. The LEMPC sends the control action computed for the first sampling period to the actuators to be implemented in a sample-and-hold fashion which is denoted as $u^*(t_k)$.

A summary of the implementation strategy for the transition from m to $m-1$ actuators is provided below:

1. At the current sampling instance t_k, the LEMPC receives a state measurement $x(t_k)$.
2. If $t_{k+N} < t_r$, go to Step 2.1. If $t_r \in [t_k, t_{k+N})$ and $(x(t_k) - x^*_{s,j}) \notin \Omega_{\rho_j}$, go to Step 2.2. Else, go to Step 2.3.
   - 2.1. If $(x(t_k) - x^*_{s,0}) \in \Omega_{\tilde{\rho}_0}$, the LEMPC (Eq. 12) operates in Mode 1. Else, the LEMPC (Eq. 12) operates in Mode 2. Go to Step 3.
   - 2.2. The LEMPC (Eq. 12) operates in Mode 2 to enforce convergence to Ω_{ρ_j}. Go to Step 3.
   - 2.3. If $(x(t_k) - x^*_{s,j}) \in \Omega_{\tilde{\rho}_j}$, the LEMPC (Eq. 13) operates in Mode 1. Else, the LEMPC (Eq. 13) operates in Mode 2. Go to Step 3.
3. The LEMPC computes its optimal input trajectory over the horizon $t \in [t_k, t_{k+N})$.
4. The LEMPC sends the optimal control action, $u^*(t_k)$ over the first sampling period ($t \in [t_k, t_{k+1})$) to the control actuators to be implemented in a sample-and-hold fashion.
5. Go to Step 1 ($k \leftarrow k+1$).

The implementation strategy for the transition from $m-1$ to m actuators is the same with the following notation changes: $0 \rightarrow j, j \rightarrow 0$, and $t_r \rightarrow t'_r$ and the control configuration switches from $m-1$ to m available actuators once the closed-loop state converges to Ω_{ρ_0}.

Remark 4. *Integrating scheduling and control is an important research topic especially in the context of Smart manufacturing albeit outside the scope of the present work. Furthermore, integrating scheduling and control is more complex than rational extensions to existing optimal control problems (e.g., MPC or EMPC) owing to the fact that there may be discrete variables in the scheduling optimal control problem making the resulting optimal control problem which integrates scheduling and control a mixed-integer nonlinear program and in general, the control horizon (i.e., the prediction horizon of MPC) is shorter than the scheduling horizon.*

Closed-loop stability analysis

In this subsection, sufficient conditions are presented such that the closed-loop state remains bounded in Ω_{ρ_0} and Ω_{ρ_j} depending on the control configuration. Previously established results of Ref. 29 are first presented for completeness of presentation. The interested reader is referred to these works for the details of these results.

Two propositions that have been previously established in Ref. 29 are presented. The first proposition bounds the difference between the nominal closed-loop trajectory ($w(t) \equiv 0$) and the actual closed-loop trajectory over one sampling period; whereas, the second proposition bounds the difference between the Lyapunov function values of two states in Ω_{ρ_i} for $i = 1, j$.

Proposition 2 (c.f. Ref. 29). *Consider the systems*

$$\begin{aligned} \dot{\hat{x}}(t) &= f(\hat{x}(t), u(0), 0), \\ \dot{x}(t) &= f(x(t), u(0), w(t)) \end{aligned} \tag{14}$$

for all $t \in [0, \Delta)$ and for any initial states $(\hat{x}(0) - x^*_{s,i}) = (x(0) - x^*_{s,i}) \in \Omega_{\rho_i}$ and $u(0) \in U_i$ for $i = 0, j$. There exists two class $\mathcal{K}$ functions $\gamma_{e,i}(\cdot)$ such that

$$|\hat{x}(\Delta)-x(\Delta)| \leq \gamma_{e,i}(\Delta) \tag{15}$$

for all $(\hat{x}(\Delta)-x^*_{s,i}), (x(\Delta)-x^*_{s,i}) \in \Omega_{\rho_i}$ *and all* $w(t) \in W$ *(for* $t \in [0,\Delta)$*) with*

$$\gamma_{e,i}(\Delta)=\frac{L_{w,i}\theta}{L_{x,i}}\left(e^{L_{x,i}\Delta}-1\right) \tag{16}$$

for $i=0,j$.

Proposition 3 (c.f. Ref. 29). *Consider the Lyapunov function* $V_i(\cdot)$ *of the system of* Eq. 1 *(with* $u \in U_i$*). There exists quadratic functions* $\alpha_{V,i}(\cdot)$ *such that*

$$V_i(x-x^*_{s,i}) \leq V_i(\hat{x}-x^*_{s,i})+\alpha_{V,i}(|x-\hat{x}|) \tag{17}$$

for all $(x-x^*_{s,i}), (\hat{x}-x^*_{s,i}) \in \Omega_{\rho_i}$ *with*

$$\alpha_{V,i}(s)=\alpha_{4,i}(\alpha_{1,i}^{-1}(\rho))s+M_{V,i}s^2 \tag{18}$$

where $M_{V,i}$ *is a positive constant for* $i=0,j$.

The following theorem provides sufficient conditions for closed-loop stability in the sense of boundedness of the closed-loop state in a compact set.

Theorem 1. *Consider the system of* Eq. 1 *in closed-loop under the proposed LEMPC design* (Eqs. 12–13) *based on a controllers* $h_0(x-x^*_{s,0})$ *and* $h_j(x-x^*_{s,j})$ *that satisfies the conditions of Eq.* 3 *for* $i=0,j$. *Let the conditions of Assumption 1 hold and let* $\epsilon_{w,i}>0, \Delta>0, \rho_i>\tilde{\rho}_i \geq \rho_{s,i}>0$ *satisfy*

$$\tilde{\rho}_i \leq \rho_i-\alpha_{V,i}(\gamma_{e,i}(\Delta)) \tag{19}$$

and

$$-\alpha_{3,i}(\alpha_{2,i}^{-1}(\rho_{s,i}))+L'_{x,i}M_i\Delta+L'_{w,i}\theta \leq -\epsilon_{w,i}/\Delta \tag{20}$$

for $i=0,j$. *If* $(x(0)-x^*_{s,0}) \in \Omega_{\rho_0}, t_N \leq t_r, t_r+t_N < t'_r$, *and* $N \geq N^* > 0$, *then, the closed-loop state is bounded in* $\Omega_{\rho_0} \cup \Omega_{\rho_j}$ *for* $t \geq 0$.

Proof. The proof consists of two main parts. First, the feasibility of the LEMPC is demonstrated for all times. Second, the main stability result (i.e., boundedness of the closed-loop state in $\Omega_{\rho_0} \cup \Omega_{\rho_j}$) is proven which is broken up into multiple subparts. The proof proceeds on the basis of the transition from m to $m-1$ actuators (i.e., the jth actuator is taken offline at t_r). Similar arguments can be made to prove similar results for the transition from $m-1$ to m actuators.

Part 1. When $t_r \notin [t_k, t_{k+N})$, the Lyapunov-based constraints of the LEMPC follow the formulation for the first phase of operation (Eq. 12). Under this phase, feasibility for any initial state $(x(t_k)-x^*_{s,0}) \in \Omega_{\rho_0}$ is guaranteed because the input trajectory obtained from the Lyapunov-based controller ($u(t_i)=h_0(\tilde{x}(t_i)-x^*_{s,0})$ for $i=k,k+1,\ldots,k+N$) is a feasible solution to the optimization problem for both Mode 1 or Mode 2 operation of the LEMPC as it satisfies the input and Lyapunov-based constraints. When $t_r \in [t_k, t_{k+N})$ and $(x(t_k)-x^*_{s,j}) \notin \Omega_{\rho_j}$, the LEMPC with the constraints of Eq. 12 operates in Mode 2. Again, the input trajectory obtained from the Lyapunov-based controller ($u(t_i)=h_0(\tilde{x}(t_i)-x^*_{s,0})$ for $i=k,k+1,\ldots,k+N$) is a feasible solution to the optimization problem as it satisfies the constraints. Lastly, if the state converges to Ω_{ρ_j} by t_r, then feasibility of the optimization problem (Eq. 13) is guaranteed because the input trajectory obtained from the Lyapunov-based controller for the control configuration with the $m-1$ actuators ($u(t_i)=h_j(\tilde{x}(t_i)-x^*_{s,j})$ for $i=k,k+1,\ldots,k+N$) is a feasible solution. Feasibility of the proposed LEMPC scheme for handling scheduled actuator maintenance hinges on boundedness inside Ω_{ρ_0} before the jth actuator is taken offline and convergence of the closed-loop state to Ω_{ρ_j} by at least the time t_r which is proven below.

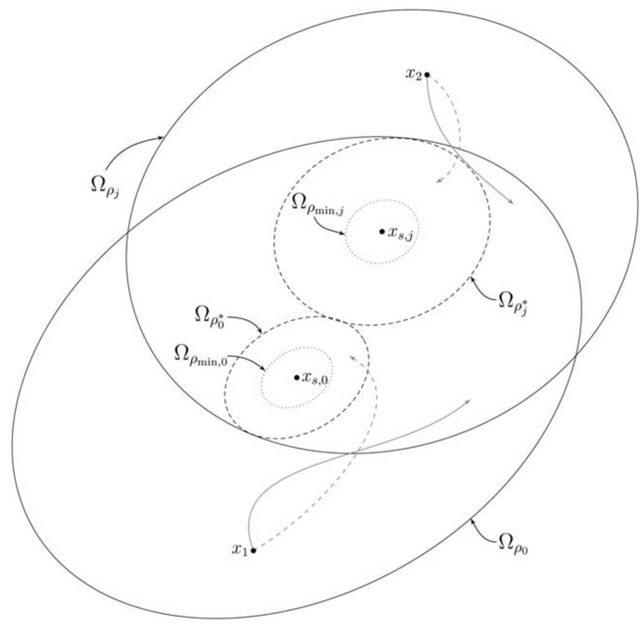

Figure 2. Illustration of the feasibility of the proposed LEMPC scheme for handling scheduled actuator maintenance.

The basic idea of feasibility of the proposed LEMPC scheme is displayed in Figure 2. Starting from x_1, one feasible input trajectory that will force the closed-loop state trajectory by t_r is the input trajectory obtained by the Lyapunov-based controller. The closed-loop state under the Lyapunov-based control input trajectory is the dashed gray line in Figure 2. However, the LEMPC forces the state to follow a trajectory (solid gray line) different because it accounts for both the process economics and the need to force the closed-loop state to Ω_{ρ_j}. A similar situation is observed in the illustration for the case when the control configuration changes from $m-1$ to m actuators (starting from x_2 in Figure 2).

Part 2.1. The proof proceeds by exploring the stability properties of Mode 1 operation of the LEMPC (Eq. 12) which occurs when $t_r \notin [t_k, t_{k+N})$ and $(x(t_k)-x^*_{s,0}) \in \Omega_{\tilde{\rho}_0}$. If $(x(t_k)-x^*_{s,0}) \in \Omega_{\tilde{\rho}_0}$, then the predicted state at the next sampling period $(\tilde{x}(t_{k+1})-x^*_{s,0}) \in \Omega_{\tilde{\rho}_0}$ as a consequence of the constraint of Eq. 12e. By Propositions 2 and 3, the following bound on the Lyapunov function can be written

$$V_0(x(t_{k+1})-x^*_{s,0})-V_0(\tilde{x}(t_{k+1})-x^*_{s,0}) \leq \alpha_{V,0}(\gamma_{e,0}(\Delta)) \tag{21}$$

which bounds the difference between the actual Lyapunov function value at the next sampling time t_{k+1} and the predicted Lyapunov function value. Because $V(\tilde{x}(t_{k+1})-x^*_{s,0}) \leq \tilde{\rho}_0$ and if $\tilde{\rho}_0$ is chosen sufficiently small such that the condition of Eq. 19 is satisfied for $i=0$, then $V_0(x(t_{k+1})-x^*_{s,0}) \leq \rho_0$ or $(x(t_{k+1})-x_{s,0}) \in \Omega_{\rho_0}$.

Part 2.2. If $(x(t_k)-x_{s,0}) \in \Omega_{\rho_0} \backslash \Omega_{\tilde{\rho}_0}$ or $t_r \in [t_k, t_{k+N})$ and $(x(t_k)-x_{s,j}) \notin \Omega_{\rho_j}$, the LEMPC operates in Mode 2 (Eq. 12) and computes a control action that satisfies

$$\frac{\partial V_0}{\partial x} f(x(t_k), u^*(t_k), 0) \leq \frac{\partial V_0}{\partial x} f(x(t_k), h_0(x(t_k)-x^*_{s,0}), 0) \quad (22)$$

The time derivative of the Lyapunov function along the state trajectory for $\tau \in [t_k, t_{k+1})$ is

$$\dot{V}_0(x(\tau)-x^*_{s,0}) = \frac{\partial V_0}{\partial x} f(x(\tau), u^*(t_k), w(\tau)) \quad (23)$$

Adding and subtracting $\dot{V}_0(x(t_k)-x^*_{s,0})$ to and from the derivative of the Lyapunov function (Eq. 23) and accounting for Eq. 3b, the time derivative of the Lyapunov function can be upper bounded over the sampling period

$$\dot{V}_0(x(\tau)-x^*_{s,0}) \leq -\alpha_{3,0}(|x(t_k)-x^*_{s,0}|) + \frac{\partial V_0}{\partial x} f(x(\tau), u^*(t_k), w(\tau)) - \frac{\partial V_0}{\partial x} f(x(t_k), u^*(t_k), 0) \quad (24)$$

for $\tau \in [t_k, t_{k+1})$. Utilizing the Lipschitz property of Eq. 5 and the bound on the disturbance, the derivative of the Lyapunov function becomes

$$\dot{V}_0(x(\tau)-x^*_{s,0}) \leq -\alpha_{3,0}(|x(t_k)-x^*_{s,0}|) + L'_{x,0}|x(\tau)-x(t_k)| + L'_{w,0}\theta \quad (25)$$

Taking into account $|f(x,u,w)| \leq M_0$ and continuity of $x(t)$, $|x(\tau)-x(t_k)|$ can be bounded for $\tau \in [t_k, t_{k+1})$ by

$$|x(\tau)-x(t_k)| \leq M_0\Delta \quad (26)$$

As $(x(t_k)-x^*_{s,0}) \in \Omega_{\rho_0} \backslash \Omega_{\tilde{\rho}_0}$ and if $\tilde{\rho}_0 \geq \rho_{s,0}$, the following bound on the deviation of the actual state and the steady-state can be derived from Eq. 3a

$$|x(t_k)-x^*_{s,0}| \geq \alpha_{2,0}^{-1}(\rho_{s,0}) \quad (27)$$

Using Eqs. 26 and 27, Eq. 25 becomes

$$\dot{V}_0(x(\tau)-x^*_{s,0}) \leq -\alpha_{3,0}(\alpha_{2,0}^{-1}(\rho_{s,0})) + L'_{x,0}M_0\Delta + L'_{w,0}\theta \quad (28)$$

for $\tau \in [t_k, t_{k+1})$. If the condition of Eq. 20 holds for $i=0$, then the Lyapunov function is decreasing along the state trajectory over the sampling period

$$\dot{V}_0(x(\tau)-x^*_{s,0}) \leq -\epsilon_{w,0}/\Delta, \quad \forall\, \tau \in [t_k, t_{k+1}) \quad (29)$$

Integrating the above bound, it is shown that

$$V_0(x(t_{k+1})-x^*_{s,0}) \leq V_0(x(t_k)-x^*_{s,0}) - \epsilon_{w,0} \quad (30)$$

for all $(x(t_k)-x^*_{s,0}) \in \Omega_{\rho_0} \backslash \Omega_{\tilde{\rho}_0}$ or $t_r \in [t_k, t_{k+N})$ and $(x(t_k)-x_{s,j}) \notin \Omega_{\rho_j}$. Using the result of Eq. 30 recursively, the state converges to $\Omega_{\tilde{\rho}_0}$ without leaving Ω_{ρ_0} in a finite number of sampling periods if $t_r \notin [t_k, t_{k+N})$.

If $t_r \in [t_k, t_{k+N})$ and $(x(t_k)-x_{s,j}) \notin \Omega_{\rho_j}$, the LEMPC operates in Mode 2 for any initial state $(x(t_k)-x^*_{s,0}) \notin \Omega_{\rho_j}$. Applying the result of Eq. 30 recursively, if the prediction horizon is chosen such that $N \geq N^*$, and if $t_N \leq t_r$ (i.e., $t_N = N\Delta$), then the state will converge to the set Ω_{ρ_j} by at least t_r. This statement holds because recursively enforcing the constraint of Eq. 12f on the computed control action will lead to successive decrease in the Lyapunov function value over each sampling period (Eq. 30) until the state converges to $\Omega_{\rho_0^*} \subseteq \Omega_{\rho_j}$ (if the conditions of Assumption 1 hold).

Part 2.3. Once the state converges to the Ω_{ρ_j}, the Lyapunov-based constraints switch to the form of Eq. 13. Applying similar steps as Part 2.1 and Part 2.2, one can show that the closed-loop is maintained in Ω_{ρ_j} if the conditions of Eqs. 19 and 20 hold for $i=j$. Also, the arguments can be repeated for the case when the control configuration switches from $m-1$ to m actuators. Therefore, the closed-loop state is maintained in $\Omega_{\rho_0} \cup \Omega_{\rho_j}$ over the length of operation when all the conditions of Theorem 1 hold. ■

Remark 5. *Regarding the economic cost function, little restrictions or assumptions are placed on it as the main stability result does not depend on the type of economic cost function used. As the economic cost is derived from the process economics, it will likely be at least a continuous function of the states and inputs. Owing to the independence of the stability result on the type of economic cost, the economic cost may change or switch when an actuator is taken offline which may happen for some applications. An issue that is not within the scope of the present work is developing an EMPC scheme that accounts for an explicitly time-varying economic cost. With traditional RTO and MPC frameworks, the economically optimal steady-state is recomputed as the process economics are updated in such a case. This will lead to different regions of operation and updated stability regions in the case of LEMPC. The interested reader is referred to* Ref. 17 *for further details on constructing a larger estimate of the stability region and explicitly handling the time-dependent economic parameters in the cost function. For the case when the estimate of the stability region is large as is often times the case when the steady-state is open-loop asymptotically stable, it may be sufficient in terms of achievable closed-loop performance to use the stability region* Ω_ρ *corresponding to one particular steady-state in the formulation of the LEMPC when the economic cost parameters change with time.*

Remark 6. *The closed-loop stability properties presented here depend on a sufficiently long prediction horizon to ensure that it is feasible to force the closed-loop state to the stability region of the next control configuration by the time the control configuration changes (i.e., the number of available actuators changes). This may lead to a computationally challenging problem to be solved online (i.e., possibly nonconvex, nonlinear optimization problem to be solved over a long horizon). To alleviate the computational burden, one could impose additional constraints on the computation of the optimal steady-state* $x^*_{s,j}$ *such that it is chosen to be close to* $x^*_{s,0}$ *so that large overlap exists between the two stability regions.*

Remark 7. *Regarding potential delay in the actuator replacement, by the time the actuator is taken offline, the closed-loop state is forced to a region in state space where stability (i.e., boundedness of the closed-loop state) can be maintained thereafter, so regardless of the time the actuator is brought back online closed-loop stability will be maintained.*

Remark 8. *If it is desirable to enforce convergence to the optimal steady-state, one can impose the Lyapunov-based constraints of either* Eq. 12f *or* Eq. 13f *depending on the* *phase of operation (i.e., the number of control actuators online) to enforce convergence to the optimal steady-state. Within this context, the provable stability is practical stability of the optimal steady-state; please refer to* Ref. 19 *for complete details on this point.*

Remark 9. *Potentially, one could extend EMPC formulated with a terminal constraint to handle actuator maintenance. Before the actuator is taken offline, the terminal constraint would be the optimal steady-state* $x_{s,0}$*. After the actuator is taken offline, the terminal constraint would be the optimal steady-state* $x_{s,j}$ *for the control system with only* $m-1$ *available actuators. When the time the actuator is taken offline is within the prediction horizon of the EMPC, the terminal constraint of the EMPC would switch from* $x_{s,0}$ *to* $x_{s,j}$*. The resulting controller would have a similar closed-loop stability property (boundedness of the closed-loop state) assuming recursive feasibility of the EMPC could be guaranteed and the stability region of the EMPC would be the feasible region of the controller. To accomplish recursive feasibility in the presence of bounded disturbances, modifications to the EMPC may be needed (e.g., use of a terminal region constraint instead of a terminal constraint, add a terminal cost etc.). Assuming these modifications can be made to guarantee recursive feasibility, one must carefully consider recursive feasibility during the transition between the two control configurations. The feasible region of EMPC formulated with a terminal constraint is difficult to explicitly characterize especially for processes with many states and inputs such as the example considered in the present work. Also, the feasible region depends on the prediction horizon. Therefore, it is difficult to guarantee that for any initial state starting in the feasible region of the first controller configuration, the state can be forced to the feasible region of the second configuration. With the proposed LEMPC, an explicit characterization of the stability region (feasible region) can be completed and it does not depend on the prediction horizon length.*

Remark 10. *The concepts presented in the present work could be extended to distributed MPC although a rigorous theoretical treatment and presentation of proposed distributed MPC algorithms for handling actuator maintenance are outside the scope of the present work. Although one could extend the distributed EMPC concepts presented in* Ref. 30 *with the concepts on merging actuator maintenance and process control presented in this work to design a distributed EMPC strategy for merging actuator maintenance and process control.*

Application to the Alkylation of Benzene Process

The proposed LEMPC for handling actuator maintenance is applied to a chemical process network example. Specifically, a process network of industrial importance used to produce ethylbenzene through the alkylation of benzene is considered. Ethylbenzene is an important chemical for the chemical process industries because it is an intermediate chemical in the production of polystyrene and other styrene-based plastics. The process flow and first-principles model of the alkylation of benzene process was first presented in Ref. 31 which was adapted from Refs. 32–35.

Description of the alkylation of benzene process

A process flow diagram of the alkylation of benzene process is displayed in Figure 3. In Figure 3, the notation F_i corresponds to the volumetric flow rate of the ith stream and the other notation will be introduced below. The process consists of five vessels: four continuously stirred tank reactors (CSTRs) and one flash tank separator (SEP-1). Benzene and ethylene, which are denoted as component A and B, respectively, are fed to CSTR-1; whereas CSTR-2 and CSTR-3 are only fed with a fresh ethylene feedstock. Within CSTR-1, CSTR-2, and CSTR-3, benzene reacts with ethylene to form the desired product, ethylbenzene, which is denoted as component C. Ethylbenzene can subsequently undergo further alkylation to form a byproduct 1,3-diethylbenzene (component D). The outlet stream of the three reactors (F_7) is fed to a flash separator to separate the desired product contained in the liquid bottoms of the separator and recover unreacted product. The overhead vapor stream containing mostly unreacted benzene of the separator is condensed and split. A portion of the condensed overhead stream is recycled back to CSTR-1. The remainder is sent to CSTR-4. Within CSTR-4, a catalyzed transalkylation reaction occurs where 1,3-diethylbenzene reacts with benzene to produce ethylbenzene. Further alkylation of ethylbenzene to 1,3-diethylbenzene also occurs in CSTR-4. To simplify the notation, the index $j=1,2,3$ is used to denote CSTR-1, CSTR-2, and CSTR-3, respectively, $j=4$ denotes SEP-1, and $j=5$ denotes CSTR-4.

The three reactions described above are considered to be the dominant reactions of the benzene alkylation process network and are summarized below

$$A+B \rightarrow C, \qquad \Delta H_{R1}=-1.5\times10^5,\ \mathrm{J/mol} \tag{R1}$$

$$B+C \rightarrow D, \qquad \Delta H_{R2}=-1.1\times10^5,\ \mathrm{J/mol} \tag{R2}$$

$$A+D \rightarrow C, \qquad \Delta H_{R3}=4.1\times10^5,\ \mathrm{J/mol} \tag{R3}$$

where R1 and R2 are exothermic and take place in CSTR-1, CSTR-2, and CSTR-3; whereas R2 and R3 (R3 is endothermic) take place in CSTR-4. The rate expression of each reaction is given by

$$r_1=0.084e^{-9502/RT}C_A^{0.32}C_B^{1.5} \tag{31}$$

$$r_2=\frac{0.085e^{-20640/RT}C_B^{2.5}C_C^{0.5}}{(1+0.015e^{-3933/RT}C_D)} \tag{32}$$

$$r_3=\frac{237.8e^{-61280/RT}C_A^{1.0218}C_D}{1+0.490e^{-50870/RT}C_A} \tag{33}$$

where C_i is the concentration of component i ($i=A,B,C,D$), R is the gas constant, and T is the absolute temperature in Kelvin.

Applying the modeling assumptions detailed in Ref. 31, a first-principles model can be constructed describing the dynamic behavior of the process network which includes 25 states (e.g., temperatures and component concentrations). Owing to the complex reaction mechanisms (Eqs. 31–33) and the recycle streams, the resulting system of coupled ordinary differential equations is a nonlinear system of the form of Eq. 1. Eight manipulated inputs are considered for the process network which includes the heat rate supplied and/or removed to each of the vessels and the inlet flow volumetric rates of the ethylene feedstock to CSTR-1, CSTR-2, and CSTR-3. The states and inputs of the process and the notation used to denote them are summarized in Table 1. The available control energy is $Q_i \in [-7.2,-0.8]\times10^6$ J/s

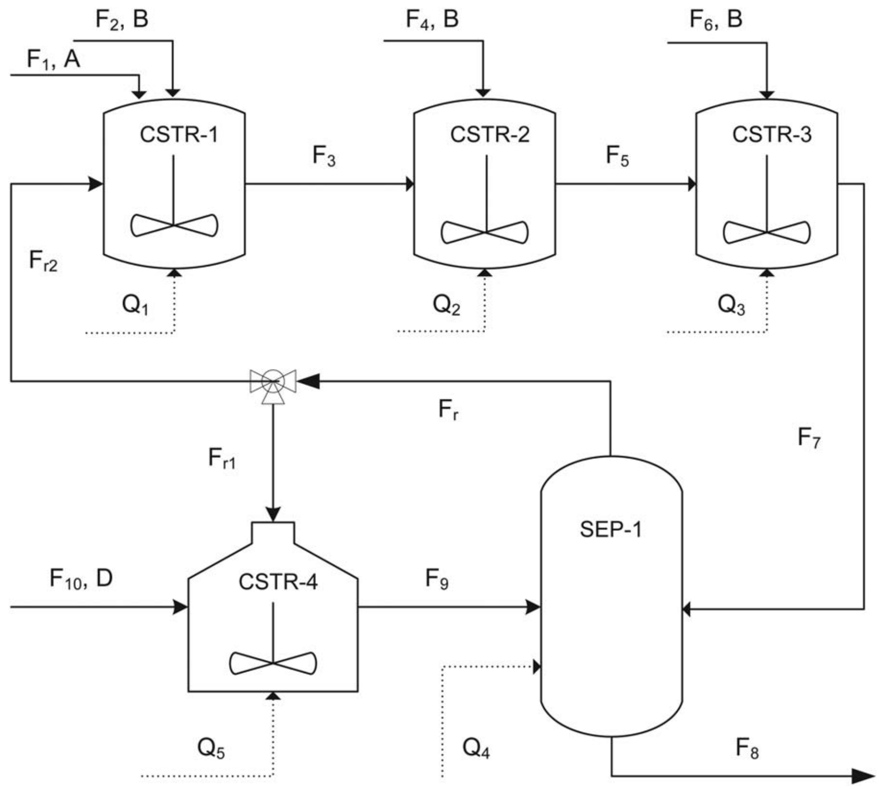

Figure 3. Process flow diagram of alkylation of benzene.

for $i=1,2,3, Q_i \in [1.6, 14.4] \times 10^6$ J/s for $i=4,5$, and $F_i \in [2.0, 18.0] \times 10^{-4}$ m^3/s for $i=2,4,6$.

Control objectives and process economics

For the benzene alkylation process, we assume that it is a priority to routinely conduct preventive maintenance on the control actuators of the process in an attempt to avoid/prevent a process upset caused by a failed or unreliable actuator. Ideally, process operators would like to maintain process operation during any maintenance task to prevent production loss as long as operators are able to sustain safe process operation with fewer control actuators. The next objective would be to operate the process network in an economically optimal fashion by accounting for the process economics and the fact that fewer actuators are available. These operating objectives are considered to be control objectives in addition to traditional control objectives (i.e., maintain process stability, robustness to disturbances etc.).

For the process network, the process economics are assumed to be adequately described by the summation of four terms

$$l_e(x,u)=L_1+L_2+L_3+L_4 \tag{34}$$

where $L_1=A_1(r_{1,1}+r_{1,2}+r_{1,3})/(r_{2,1}+r_{2,2}+r_{2,3})$ is the weighted instantaneous selectivity of reaction R1 over reaction R2 in CSTR-1, CSTR-2, and CSTR-3 (the notation $r_{i,j}$ denotes the ith reaction rate in the jth vessel), $L_2=A_2r_{3,4}$ is the weighted production rate of C through R3 in CSTR-4, $L_3=A_3F_8C_{C4}$ is the weighted (instantaneous) molar flow rate of the desired product out of the separator, and L_4 is given by the following expression

$$L_4=A_4\sum_{i=1}^{3}Q_i-A_5\sum_{i=4}^{5}Q_i \tag{35}$$

which penalizes energy consumption and removal from the process network. The coefficients $A_k, k=1,2,3,4,5$ are positive weighing factors with the following values: $A_1=1.00$, $A_2=1.50, A_3=0.50, A_4=1.25\times10^{-6}$, and $A_5=2.00\times10^{-6}$. The reason for the difference between the weight values A_4 and A_5 is associated with the difference in cost of removing heat from a vessel and cost of supplying heat to a vessel.

Additionally, the amount of ethylene that may be fed to each CSTR over the operating horizon t_f is constrained to be equal to an average amount

$$\frac{1}{t_f}\int_{t_0}^{t_0+t_f}F_i(\tau)C_{Bi0}d\tau=F_{\text{avg},i}C_{Bi0}, \quad i=2,4,6 \tag{36}$$

where C_{Bi0} denotes the concentration of ethylene in the ith stream $(i=2,4,6)$ and $F_{\text{avg},i}$ is the average flow rate amount of the ith stream. In the example below, $F_{\text{avg},i}$ is taken to be a steady-state flow rate. The three economics-oriented constraints formed from Eq. 36 allow for the LEMPC to distribute ethylene in a nonuniform fashion (with respect to time)

Table 1. Process State and Input Variables

$C_{A1}, C_{B1}, C_{C1}, C_{D1}$	Concentrations of A, B, C, D in CSTR-1
$C_{A2}, C_{B2}, C_{C2}, C_{D2}$	Concentrations of A, B, C, D in CSTR-2
$C_{A3}, C_{B3}, C_{C3}, C_{D3}$	Concentrations of A, B, C, D in CSTR-3
$C_{A4}, C_{B4}, C_{C4}, C_{D4}$	Concentrations of A, B, C, D in SEP-1
$C_{A5}, C_{B5}, C_{C5}, C_{D5}$	Concentrations of A, B, C, D in CSTR-4
T_1, T_2, T_3, T_4, T_5	Temperatures in each vessel
F_2, F_4, F_6	Ethylene feedstock flow rates
Q_1, Q_2, Q_3, Q_4, Q_5	External heat supply/removal to each vessel

Table 2. Steady-State Input Values for the Benzene Alkylation Process Network

Q_{1s}, Q_{2s}, Q_{3s}	-4.0×10^6 J/s
Q_{4s}, Q_{5s}	8.0×10^6 J/s
F_{2s}, F_{4s}, F_{6s}	1.0×10^{-3} m^3/s

to each reactor if it is economically desirable. However, the LEMPC must compute an input profile that uses the same amount of ethylene as the amount used when ethylene is distributed (fed) uniformly with time.

LEMPC design

A control scheme that can achieve the control objectives outlined above is the proposed LEMPC for handling preventive maintenance of control actuators for the benzene alkylation process network. In this subsection, the design of the LEMPC is detailed and subsequently, applied to the benzene alkylation process network. The benzene alkylation process network is modeled by an input-affine nonlinear system, that is, the vector field f of Eq. 1 has a specific form

$$f(\bar{x}(t), u(t), 0) = \bar{f}(\bar{x}(t)) + \sum_{j=1}^{8} \bar{g}_i(\bar{x}(t)) u_i(t) \tag{37}$$

where $\bar{f}: \mathbb{R}^{25} \to \mathbb{R}^{25}$ and $\bar{g}_i: \mathbb{R}^{25} \to \mathbb{R}^{25}$ for $i=1,\ldots,8$. For simplicity of presentation, we assume that after converting the system of Eq. 37 into appropriate deviation variables, the origin is the steady-state of Eq. 37 and we drop the $\bar{\cdot}$ notation for the remainder. To design a Lyapunov-based controller for the process network, the input vector is partitioned on the basis of the input type: $u=[u_h \; u_f]^T$ where u_h denotes a vector with the heat rate inputs ($Q_i, i=1,2,3,4,5$) and u_f denotes a vector with the flow rate inputs ($F_i, i=2,4,6$). The reason for partitioning the input vector in this fashion is because the heat rate inputs are considered to have the most influence on maintaining process stability and the flow rates are essentially additional degrees of freedom that can be used to optimize the process economics and/or be used to compensate for fewer online actuators. The Lyapunov-based controller designed to asymptotically stabilize the origin of the nonlinear system is designed element-wise to reflect the difference in responsibilities of the heat rate inputs and the flow rate inputs. For the heat rate inputs, the following feedback control law[36] is used

$$h_{hi}(x) = \begin{cases} -\dfrac{L_f V + \sqrt{(L_f V)^2 + (L_{g_i} V)^4}}{(L_{g_i} V)^2} L_{g_i} V & \text{if } L_{g_i} V \neq 0 \\ 0 & \text{if } L_{g_i} V = 0 \end{cases} \tag{38}$$

for $i=1,\ldots,5$ where $L_f V = \frac{\partial V}{\partial x} f(x)$ and $L_{g_i} V = \frac{\partial V}{\partial x} g_i(x)$ denote the Lie derivatives of V with respect to the vector fields f and g_i, respectively. For the flow rate inputs, the Lyapunov-based controller elements are $h_f(x) = [0\,0\,0]^T$ (in appropriate deviation variables). Thus, the resulting Lyapunov-based controller is $h(x) = [h_h^T(x) \; h_f^T(x)]^T$.

To characterize the stability region $\Omega_{\tilde{\rho}_0}$, a steady-state is iteratively selected so that it is within the acceptable operating range and its corresponding stability region is a region in state space where state and input constraints are satisfied. The steady-state denoted as $x_{s,0}$ that satisfies these conditions corresponds to the steady-state input values contained in Table 2. On converting to appropriate deviation variables, $x_{s,0}$ is taken to be the origin in the deviation state-space coordinates. A quadratic Lyapunov function (i.e., $V(x) = x^T P x$ where P is a positive definite matrix) is considered with

$$P = \text{diag}([1\;10\;1\;1\;100\;1\;10\;1\;1\;100\;1\;10\;1\;1\;100\;1\;10\;1\;1\;100\;1\;10\;1\;1\;100])$$

which has been selected such that the closed-loop alkylation benzene process network under the Lyapunov-based controller $h(x)$ gives a desirable closed-loop response with respect to traditional control objectives (i.e., speed of response, fewer oscillations, etc.). Given that the steady-state $x_{s,0}$ is open-loop asymptotically stable, the stability region $\Omega_{\tilde{\rho}_0}$ is estimated to be a large region in state space with $\tilde{\rho}_0 = 1.0 \times 10^8$.

Nominal operation ($w(t) \equiv 0$) of the benzene alkylation process is considered below for the case studies. Therefore, only Mode 1 operation of the LEMPC is used, and the Lyapunov-based constraint of the LEMPC is formulated on the basis of the steady-state $x_{s,0}$. The sampling time of the LEMPC is chosen to be $\Delta = 20$s and the operating period that the material constraint of Eq. 36 is enforced is $t_f = 1800\,\text{s} = 30\,\text{min}$. To solve the optimization problem of the LEMPC at each sampling period, Ipopt[37] is used.

Before we proceed with applying the LEMPC to demonstrate its capability of handling actuator maintenance, we apply it to the process network to show its applicability on closed-loop economic performance improvement over operating the process network at the economically optimal steady-state. Specifically, the steady-state optimization problem is solved using the steady-state model. The steady-state optimization problem is given by

$$\max_{x_s, u_s} \; l_e(x_s, u_s) \tag{39a}$$

$$\text{s.t. } f(x_s) + \sum_{j=1}^{8} g_i(x_s) u_s = 0 \tag{39b}$$

$$u_s \in U_0 \tag{39c}$$

$$x_s \in X_s \tag{39d}$$

$$V(x_s) \leq \tilde{\rho}_0 \tag{39e}$$

where $l_e(x_s, u_s)$ is the economic cost function of Eq. 34 and $f(x_s, u_s, 0) = 0$ is the steady-state process model for the benzene alkylation process network. The optimal steady-state, denoted as $x^*_{s,0}$ corresponds to the steady-state input $u^*_{s,0}$ given in Table 3.

Table 3. Optimal Steady-State Input Values with Respect to the Economic Cost Function of Eq. 34 and All Inputs Online

Q^*_{1s}	-8.0×10^5 J/s	Q^*_{5s}	9.4×10^6 J/s
Q^*_{2s}	-8.1×10^5 J/s	F^*_{2s}	5.5×10^{-4} m^3/s
Q^*_{3s}	-8.0×10^5 J/s	F^*_{4s}	3.2×10^{-4} m^3/s
Q^*_{4s}	3.1×10^6 J/s	F^*_{6s}	2.6×10^{-4} m^3/s

In the optimization problem of Eq. 39, the constraint of Eq. 39c is the available control energy with all available actuators online. The constraint of Eq. 39d enforces that the computed optimal steady-state be in the set of admissible steady-states and is defined through five temperature constraints

$$|T_{js} - T_{js,0}| \leq 0.1 T_{js,0} \tag{40}$$

for $j=1,\ldots,5$ where T_{js} is the steady-state temperature of the jth vessel (i.e., a decision variable of Eq. 39) and $T_{js,0}$ is

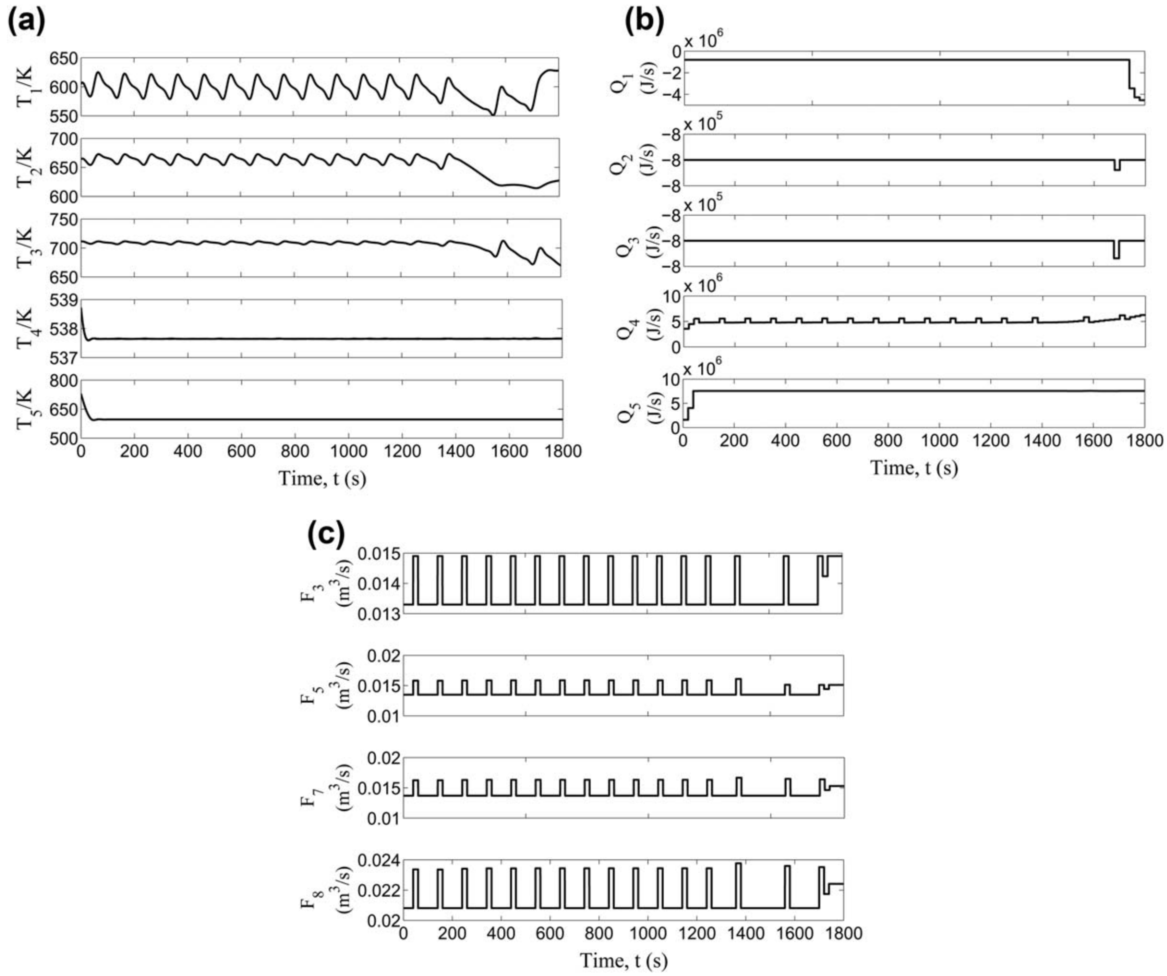

Figure 4. Closed-loop (a) temperature, (b) heat rate, and (c) flow rate profiles under LEMPC with $N = 2$.

the steady-state temperature of the jth vessel corresponding to the steady-state $x_{s,0}$. Lastly, the constraint of Eq. 39e is a Lyapunov-based constraint to ensure the optimal steady-state is contained in the stability region $\Omega_{\tilde{\rho}_0}$. Given that $\Omega_{\tilde{\rho}_0}$ is a large region in state space that satisfies process constraints, this region will be used in the LEMPC formulation. For a fair comparison between the closed-loop performance under LEMPC and steady-state operation, the optimal steady-state must be a reachable steady-state when the LEMPC is operated in LEMPC Mode 1. The Lyapunov-based constraint of the LEMPC (Eq. 11e) that enforces the predicted state be inside of $\Omega_{\tilde{\rho}_0}$, which is formulated with the steady-state $x_{s,0}$ (i.e., not necessarily the optimal steady-state), must therefore contain the optimal steady-state $x^*_{s,0}$.

We compare the closed-loop performance of the benzene alkylation process under LEMPC (Mode 1 operation of the LEMPC only) and steady-state operation over one period of operation (operating window of 30 min). The material constraint of Eq. 36 is added to formulation of the LEMPC of Eq. 11 and the optimal flow rates are used as $F_{\text{avg},i}$ (i.e., $F_{\text{avg},i} = F^*_{is}$ for $i = 2, 4, 6$). The benzene alkylation process is initialized at the economically optimal steady-state $x^*_{s,0}$. For the case of steady-state operation, the total economic cost is computed by maintaining operation at the steady-state over the period of operation considered. The total economic cost is defined as

$$J = \int_0^{t_f} l_e(x(\tau), u(\tau)) d\tau \tag{41}$$

where $l_e(\cdot,\cdot)$ is the economic cost function of the benzene alkylation process of Eq. 34. The following prediction horizons of the LEMPC were considered: $N = 2, 4, 6, 8, 10$.

Figures 4 and 5 display the temperature, heat rate, and flow rate profiles of the closed-loop benzene alkylation process under LEMPC with a prediction horizon of $N = 2$ and $N = 6$, respectively. For the cases with $N > 6$, no significant performance improvement was observed with respect to the total economic cost of Eq. 41 over the case with $N = 6$. With $N = 2$, LEMPC operates the process network in a cyclic operating pattern (Figure 4) and exhibits three phases of operation over the operating window which is observed in the temperature profiles of Figure 4a. The first phase from approximately $t = 0$ to $t = 80$ s is caused by the effect of initial condition; the second phase from approximately $t = 80$ s to $t = 1400$ s is a cyclic operating pattern; the last phase results from the material constraint as it needs to be satisfied over the operating window (refer to Figure 4c). With $N = 6$ (Figure 5), a similar, albeit chaotic-like, behavior is observed. The chaotic-type operation is mainly associated with the computed input profiles Q_4 and Q_5 which could be due to the interaction between the two inputs (i.e., different Q_4 and Q_5 combinations yield similar economic cost values)

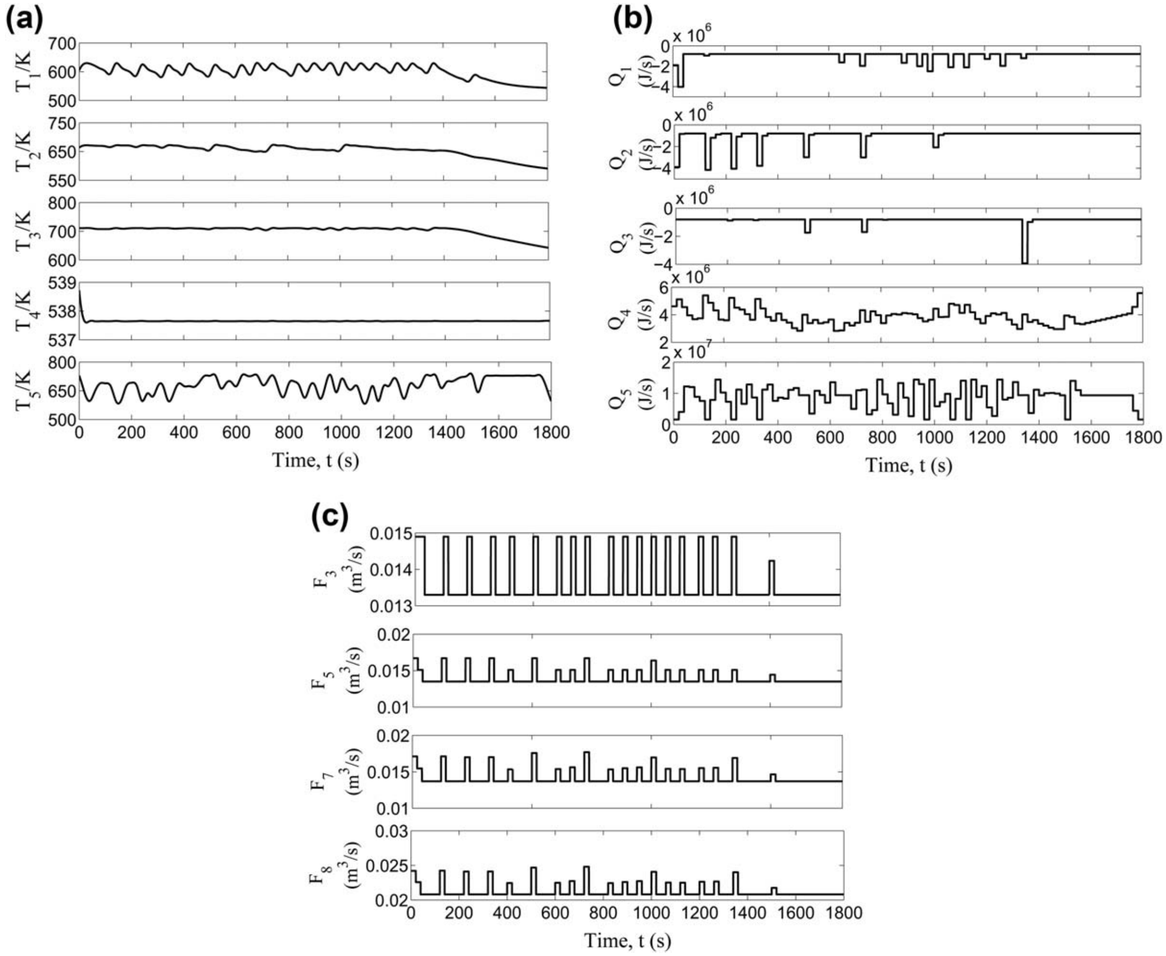

Figure 5. Closed-loop (a) temperature, (b) heat rate, and (c) flow rate profiles under LEMPC with $N = 6$.

and/or the cost is not particularly sensitive to these inputs. However, it is important to point out that the operation pattern enforced by the computed input profile of the LEMPC is not truly chaotic because repeated simulations with $N = 6$ yielded the same closed-loop profiles.

The LEMPC is applied to the process network with several prediction horizons: $N = 2$, 4, 6, 8, 10 to consider the effect of the prediction horizon length on closed-loop economic performance. As shown in Figure 6, the closed-loop economic performance increases with longer prediction horizon. For the benzene alkylation process under the LEMPC with $N = 2$, the total economic cost is 12.01% over steady-state operation; while the total economic cost is 15.92% greater under the LEMPC with $N = 6$ compared to steady-state. From this analysis, nonuniform (with respect to time) distribution of ethylene to the benzene alkylation process network yields greater total economic cost than uniform in time distribution of ethylene to the process network. Even though the benzene alkylation process is operated in a nonperiodic dynamic operating pattern, $N = 6$ was selected as the appropriate prediction horizon of the LEMPC to use in the case studies considered below.

Remark 11. *If one deemed the operating pattern enforced by the LEMPC with $N = 6$ is undesirable, one could potentially add penalty terms in the cost function of the LEMPC penalizing the rate of change of the inputs (i.e., add a quadratic term of the form $(u(t_{k+1})-u(t_k))^T R_c (u(t_{k+1})-u(t_k))$ to the cost where R_c is a positive definite matrix) so that the LEMPC computes a smoother input profile.*

LEMPC for handling actuator maintenance

We apply the proposed LEMPC (Eqs. 12–13) to the benzene alkylation process network. The ethylene material

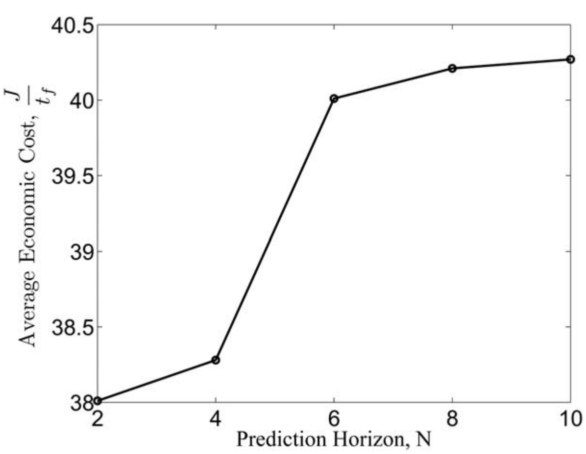

Figure 6. The effect of the prediction horizon on closed-loop economic performance under LEMPC.

Table 4. Optimal Steady-State Input Values with Respect to the Economic Cost Function of Eq. 34 with the Q_4 Actuator Taken Offline

Q^*_{1s}	-6.4×10^5 J/s	Q^*_{5s}	6.7×10^6 J/s
Q^*_{2s}	-12.3×10^5 J/s	F^*_{2s}	3.1×10^{-4} m^3/s
Q^*_{3s}	-8.0×10^5 J/s	F^*_{4s}	6.8×10^{-4} m^3/s
Q^*_{4s}	8.0×10^6 J/s	F^*_{6s}	4.5×10^{-4} m^3/s

constraint (Eq. 36) is added as a constraint in the LEMPC of Eq. 11 based on the optimal flow rates $F_{avg,i}=F^*_{is}$ for $i=2,4,6$ which are given in Table 3. For the following case studies, we consider preventive maintenance in the actuator supplying heat to the separator. We assume that the Q_4 ($j=4$) actuator loses its ability to change and becomes fixed at its steady-state value ($Q_{4s}=8.0\times10^6$ J/s). To motivate the practical scenario for this, the heat supplied to the separator could be provided through a steam jacket and the maintenance is scheduled for the flow control valve that controls the steam pressure. To perform the maintenance task, we assume that another (uncontrolled) steam line can provide steam to the jacket at a constant rate. As a consequence, the steady-state when all available actuators are online and when the Q_4 actuator is taken offline are the same.

Case I: Actuator Taken Offline for Preventive Maintenance. We consider a preventive maintenance task will be completed on the actuator that manipulates the amount of heat supplied to the separator Q_4. The actuator is scheduled to be shut down at $t_r=400$ s for a preventive maintenance task to be completed on it. The optimal steady-state is computed for the case with the Q_4 actuator taken offline and is given in Table 4. From the theoretical developments, the

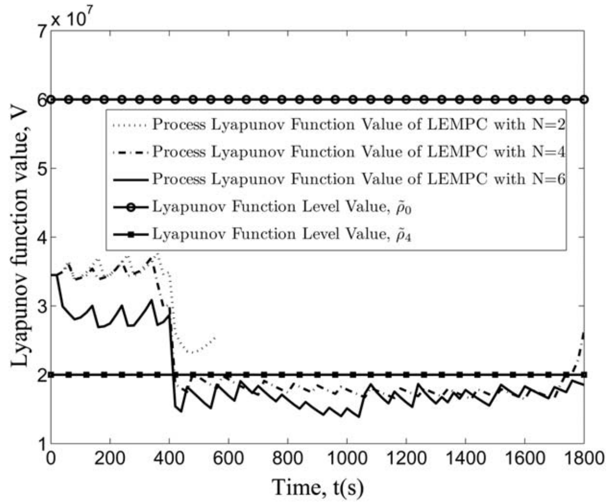

Figure 7. Impact of LEMPC prediction horizon on process stability.

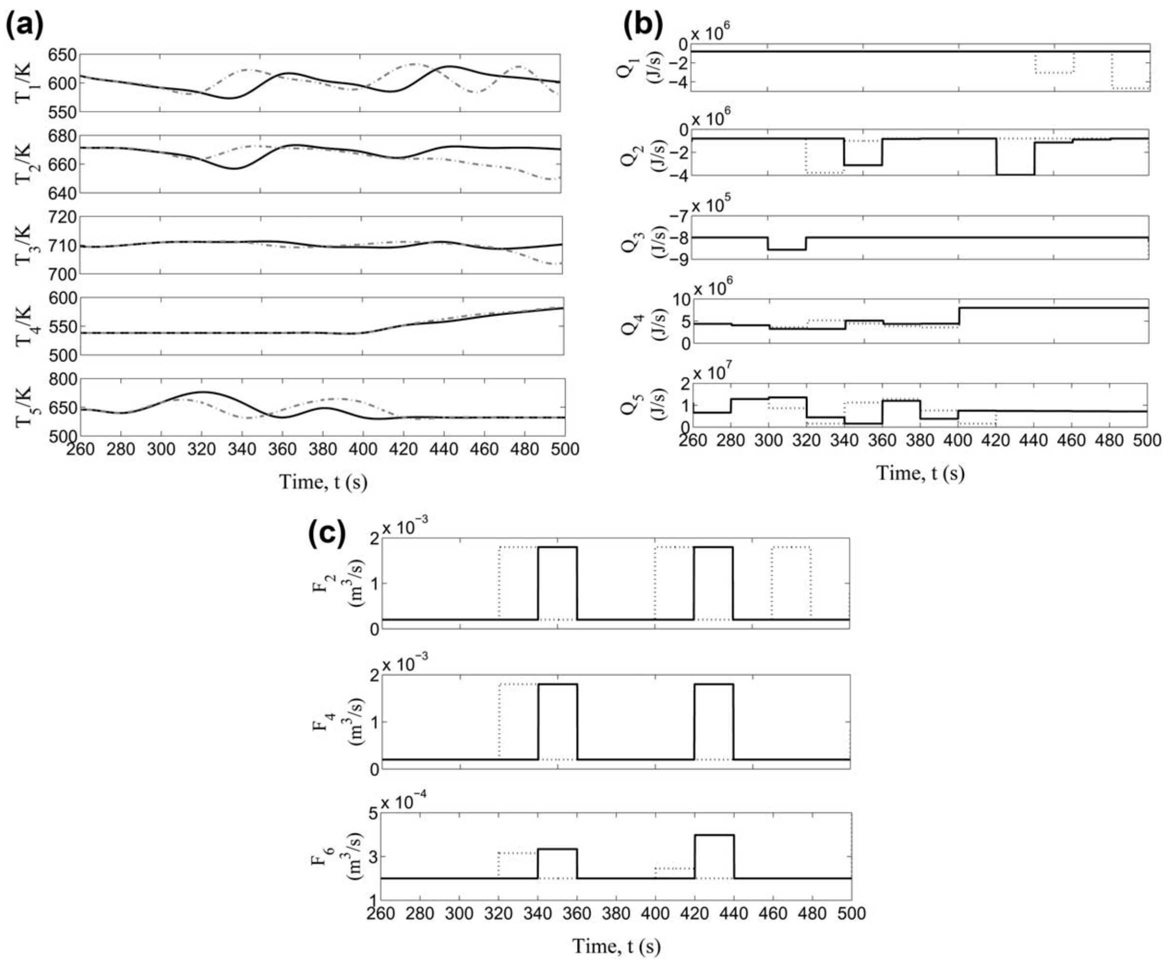

Figure 8. Closed-loop (a) temperature, (b) heat rate, and (c) flow rate profiles under the proposed LEMPC for handling actuator maintenance (solid line) and under LEMPC-A (dashed line).

The Q_4 actuator is taken offline at $t_r=400$ s.

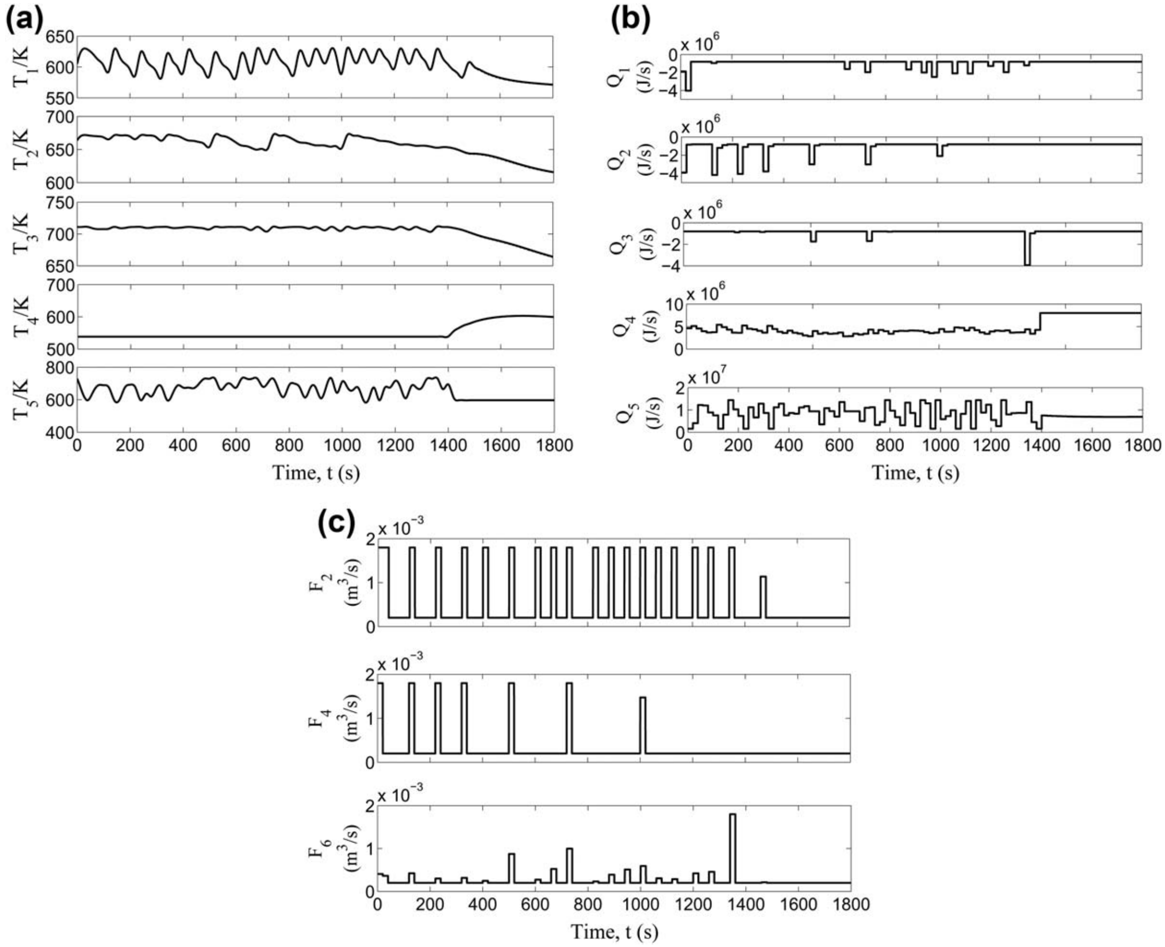

Figure 9. Closed-loop (a) temperature, (b) heat rate, and (c) flow rate profiles under the proposed LEMPC (Eqs. 12–13 with Mode 1 only).

The Q_4 actuator is taken offline at $t_r = 1400$ s.

prediction horizon should be sufficiently long to ensure feasibility and stability of the closed-loop process network ($N \geq N^*$). We investigate this property in a series of closed-loop simulations of the chemical process network under the proposed LEMPC. The proposed LEMPC is applied to the chemical process network with prediction horizons: $N=2, 4, 6$. The Mode 1 Lyapunov-based constraints of Eqs. 12e and 13e are based on the economically optimal steady-state corresponding to the steady-state input given in Tables 3 and 4 for all available actuators and with the Q_4 actuator taken offline, respectively. The value of the level set of the Lyapunov function where time-varying operation is allowed is $\tilde{\rho}_0 = 6.0 \times 10^7$ and $\tilde{\rho}_4 = 2.0 \times 10^7$ for each of the two cases, respectively. The Lyapunov function values of these simulations are given in Figure 7.

From Figure 7, the LEMPC of Eqs. 12–13 with prediction horizon $N = 2$ fails to force the closed-loop state into $\Omega_{\tilde{\rho}_4}$ by t_r resulting in the LEMPC problem becoming infeasible. The prediction horizon $N=4$ is sufficiently long to force the closed-loop state to $\Omega_{\tilde{\rho}_4}$ by t_r. However, at the end of the operating period, the closed-loop state goes outside $\Omega_{\tilde{\rho}_4}$ owing to the need to satisfy the input material constraint at the end of the operating period. For a prediction horizon $N = 6$, the closed-loop simulation confirms that this is a sufficiently long horizon to overcome the two sources of infeasibility. Based on both closed-loop stability and performance (discussed above), a prediction horizon $N = 6$ is used for all of the subsequent closed-loop simulations.

For the remainder of the work, the steady-state and level set used in the formulation of the Mode 1 Lyapunov-based constraint of the LEMPC are $x_{s,0}$ (i.e., the one corresponding to the steady-state input values contained in Table 2) and $\tilde{\rho}_0 = 1.0 \times 10^8$, respectively, because this is a steady-state for both all available actuators are online and the Q_4 actuator is taken offline. In the next simulation, we demonstrate the benefit of integrating actuator maintenance, process economics, and process control in a unified system. The closed-loop temperature and input profiles are given in Figure 8. We compare the closed-loop behavior of the benzene alkylation process under the proposed LEMPC (Eqs. 12–13) with two other cases under different LEMPC formulations. In the first case, denoted as LEMPC-A, the LEMPC uses the input constraint with all available control actuators online until the sampling period $t_r = 400$ s where the input constraint switches to the remaining available actuators. For this case, LEMPC-A cannot proactively prepare for when the Q_4 actuator will be taken offline because LEMPC-A does not account for the control system change until after the actuator is taken offline. However, it is unlikely that current control systems even have the ability to easily change from m to $m-1$ actuators. Therefore, a second case, denoted as LEMPC-B, was considered where the LEMPC continues to compute

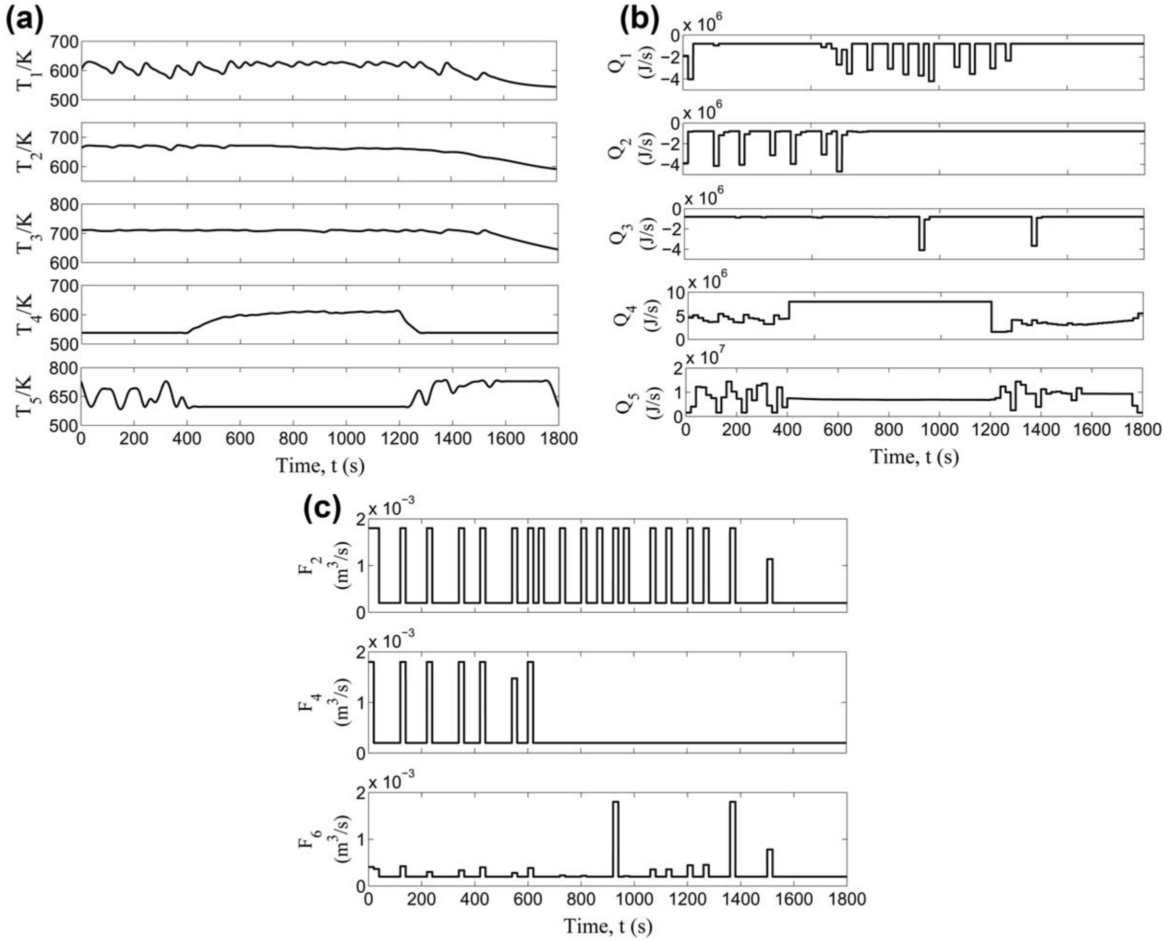

Figure 10. Closed-loop (a) temperature, (b) heat rate, and (c) flow rate profiles under the proposed LEMPC (Eqs. 12–13 with Mode 1 only).

The Q_4 actuator is briefly taken offline for inspection from $t_r = 400$ s to $t'_r = 1200$ s.

control actions for all the inputs even after the Q_4 actuator is taken offline. This second case may be considered as the worst case scenario. The closed-loop profiles of the process network under LEMPC-A are also shown in Figure 8 as the dashed profiles. The most noticeable difference between the two profiles is that the temperature for CSTR-4 (T_5) under the proposed LEMPC is less than that the one under LEMPC-A when Q_4 is taken offline. Comparing the total economic cost of the three scenarios, the total economic cost under the proposed LEMPC is 2.17% higher than that of the process under LEMPC-A; the total economic cost under the proposed LEMPC is 5.38% greater than that under LEMPC-B.

To demonstrate that the LEMPC can maintain stability of the process regardless of the time the actuator is taken offline (i.e., the choice of t_r is arbitrary), we perform another simulation where the Q_4 actuator is taken offline at $t_r = 1400$ s. The closed-loop profiles of this case are shown in Figure 9. The LEMPC is able to maintain boundedness of the closed-loop state as observed from the temperature profile of Figure 9a.

Case II: Actuator Briefly Taken Offline for Inspection. Another important part of scheduled preventive maintenance is routine inspection of operating equipment. For this case, we consider that the Q_4 actuator is briefly taken offline for a routine inspection. The time the actuator is taken offline is $t_r = 400$ s and after the inspection is completed, the actuator is brought back online at $t_r' = 1200$ s. The temperature and input profiles of this closed-loop simulation are displayed in Figure 10. Again, stability throughout the simulation is maintained under the LEMPC.

Case III: Multiple Actuators Taken Offline for Inspection. Within the context of hybrid or switched systems minimum dwell time is important as sufficiently fast switching between modes of operation may cause the closed-loop system to become unstable. Within the context of the present work, the issue of stability after taking an actuator offline has more to do with the controllability of the system and not the time the actuators are taken offline. In other words, before any actuator is taken offline, the closed-loop state trajectory is forced to a set in state space where stability (i.e., boundedness of the closed-loop state) can be maintained with the remaining actuators which is imposed through the constraints of the proposed LEMPC. If the closed-loop state cannot be forced to this region in state space with the available control energy or if no such region exists (i.e., when multiple actuators are offline), one may adopt a maintenance policy where only one actuator can be taken offline for maintenance at a time. One would expect that this type of maintenance policy would not pose many practical restrictions considering the limited availability of resources to accomplish these maintenance tasks such as limited maintenance personnel.

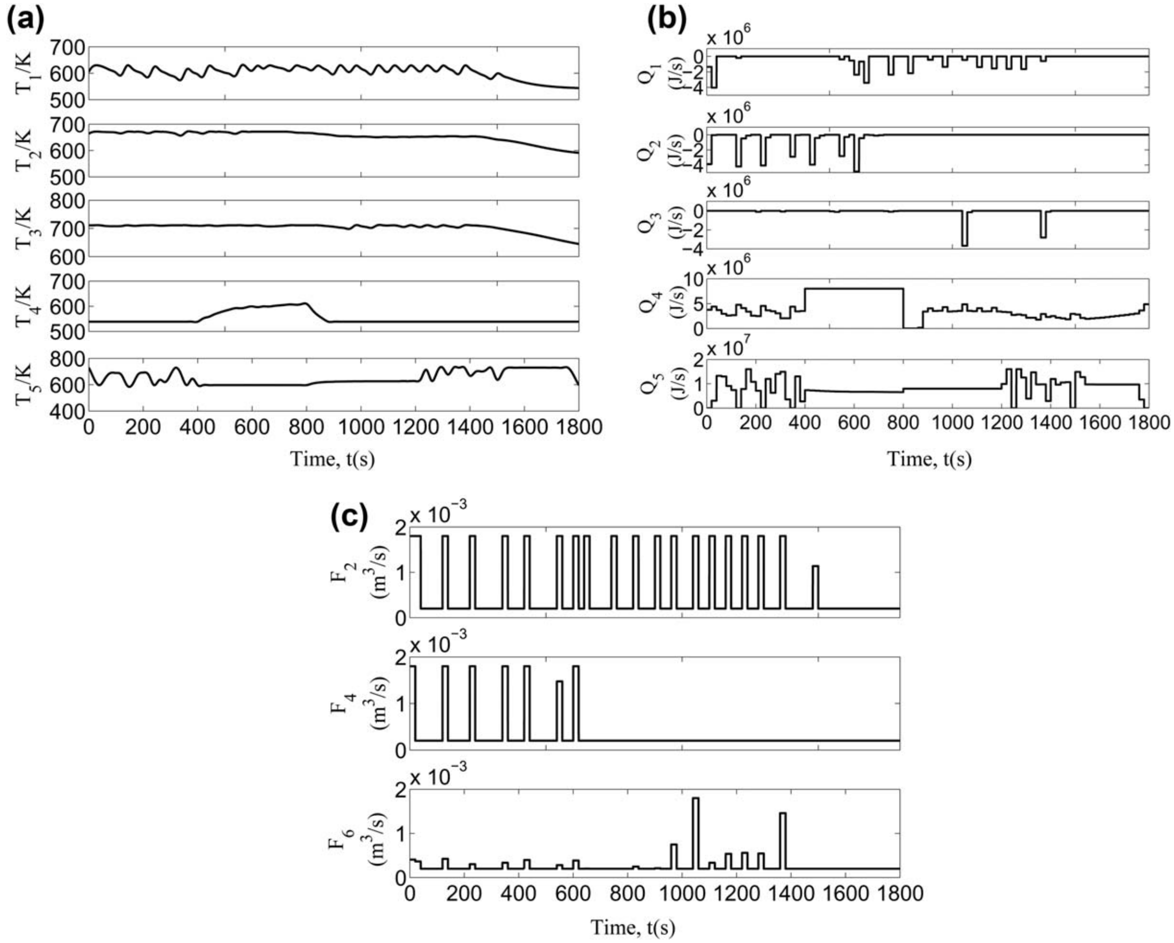

Figure 11. Closed-loop (a) temperature, (b) heat rate, and (c) flow rate profiles under the proposed LEMPC (Eqs. 12–13 with Mode 1 only).

The Q_4 actuator is briefly taken offline for inspection from $t_{r1} = 400$ s to $t'_{r1} = 800$ s and the Q_5 actuator is briefly taken offline for inspection from $t_{r2} = 800$ s to $t'_{r2} = 1200$ s.

To demonstrate this point, consider that the Q_4 actuator is taken offline for a routine inspection between $t_{r1} = 400$ s and $t'_{r1} = 800$ s and the Q_5 actuator is taken offline for a routine inspection just after the Q_4 actuator is brought back online at $t_{r2} = t'_{r1} = 800$ s and be available again at $t'_{r2} = 1200$ s after its repair. The temperature and input profiles of this closed-loop simulation are displayed in Figure 11. Once the Q_4 is taken offline, a noticeable increase in the temperature of the separator (T_4) is observed (Figure 11a) and similarly, for the temperature of CSTR-4 (T_5) after the Q_5. However, stability of the closed-loop system was maintained throughout.

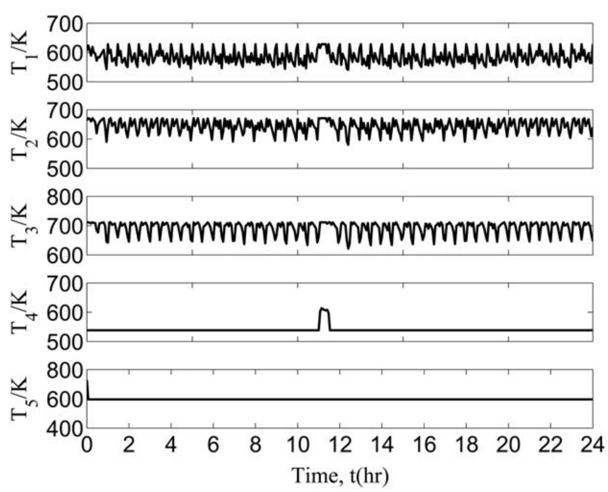

Figure 12. Closed-loop temperature profiles under the proposed LEMPC (Eqs. 12–13 with Mode 1 only with time-varying economic cost function).

The Q_4 actuator is briefly taken offline for inspection from $t_r = 11.5$ h to $t'_r = 12.0$ h.

Case IV: Actuators Inspection Under Time-Varying Economic Cost. In industry, the energy cost may change frequently because of variable electric power demand. Based on this consideration, we consider the case that a weight may be explicitly time-varying in economic cost. Following a realistic electricity price trend in a single day,[38] we assume the parameter A_5 increases 3.33% per half an hour for the first 12 h and then decreases 3.33% per half an hour for the second 12 h. The Q_4 actuator is taken offline for a routine inspection between $t_r = 11.5$ h and $t'_r = 12.0$ h. The temperature profiles of a closed-loop simulation are displayed in Figure 12. Stability of the closed-loop system is maintained throughout the 24-h length of operation.

Conclusions

This work focused on the development of a LEMPC to integrate preventive maintenance of control actuators, process economic performance, and process control. During a scheduled preventive maintenance task on the *j*th control actuator, the actuator is effectively taken offline. In general, the steady-state with all available control actuators and with actuators taken offline to perform a scheduled preventive maintenance task may be different (i.e., the former may not even be a steady-state of the latter scenario). To address this point, the proposed LEMPC was designed to ensure that the closed-loop state will be forced from the stability region of the steady-state of all m actuators to the stability region with $m-1$ actuators online before the *j*th control actuator was taken offline. Closed-loop stability in the sense of boundedness of the closed-loop state was proved. The LEMPC capable of handling preventive maintenance was applied to a benzene alkylation process which yielded improved closed-loop economic performance over steady-state operation and demonstrated its ability to handle changing number of online actuators.

Acknowledgment

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